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Hawking Tunneling Radiation of Global Monopole Charged Black Hole in Lorentz Invariance Violating Scalar Field

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Abstract The scalar field equation based on Lorentz invariance violation is generalized to curved space-time, and is corrected by aether-like terms. Then the modified Hamilton-Jacobi equation is obtained under the condition of semi-classical approximation, and by which the characteristics of Hawking tunneling radiation of global monopole charged black hole are researched. The results show that the effects of aether-like terms may increase the temperature and decrease the entropy of the black hole compared to before correction. This work can also help to understand the properties of Lorentz invariance violation in curved space-time.

Keywords: Modified scalar field equation, Hamilton-Jacobi equation, Hawking radiation, corrected entropy.

1 Introduction

Classical black holes cannot radiate energy or particles outward. In the 1970s Hawking found that quantum radiation could be emitted from the event horizon of black holes, which was called Hawking radiation[1,2]. This discovery soon led to a series of effective studies on the thermal radiation of stationary and non-stationary black holes [3,4,5,6,7,8,9,10]. Subsequently, Kraus, Parikh, Wilczek et al proposed the theory of energy quantum tunneling, believing that virtual particles inside the black hole penetrate the event horizon through the quantum tunneling effect, forming real particles shooting outside and forming Hawking radiation. The temperature and entropy of Hawking radiation were calculated, and the pure thermal radiation theory of Hawking radiation was modified [11,12].

In 1999, the semi-classical Hamilton-Jacobi equation (later referred to as H-J equation) was proposed to study the tunneling radiation of black holes, after which people could easily calculate the tunneling probability and Hawking temperature of scalar particles [13]. Subsequently, people studied the quantum tunnel behavior of various particles near the event horizon by using H-J equation [14,15,16,17,18,19,20,21,22,23,24], among whom Lin and Yang studied the motion of fermions and bosons in curved space-time and obtained various particles' motion equations – H-J equations. H-J method has found the internal connection among the Dirac equation, Rarita-Schwinger equation, Klein-Gordon equation and the passive Maxwell field equation, reflects the Lorentz invariance of the theories[23,24]. However, the Lorentz invariance needs to be modified in high energy field inferred by the research of quantum gravity theory, which is so called Lorentz invariance violation [25]. In 2019, Yang, Lin and Pu extended the Lorentz-violating scalar and vector field equation to curved space-time, and the further modified H-J equation was obtained under the semi-classical approximation, which can help us better understand the physical properties of Lorentz invariance violation effect in curved space-time[26,27].

The most innovative aspect of current research on static, stationary and non-stationary black holes is the correction of tunneling radiation. There are no more reports on the effect of Lorentz modification on tunneling under high energy conditions. In this paper, we will research the Hawking radiation characteristics of the global monopole charged black hole by the modified H-J equation in Lorentz-violating scalar field. The structure of this paper is as follows. In section 2, we will give the modified H-J equation in a scalar field in curved space-time based on Lorentz invariance violation. Section 3 will calculate the corrected tunneling probability, Hawking temperature and entropy of the global monopole charged black hole according to the modified H-J equation. Section 4 will give the research conclusion of this paper.

2 Lorentz-violating Scalar Field Equation and Modified Hamilton-Jacobi Equation

In the high energy field, the study on quantum gravity infers that Lorentz invariance needs to be modified on the Planck scale. In recent years, a theory of Lorentz scalar field has been proposed, and its action can be written as [25]

$$L = \frac{1}{2}[\partial_\mu\varphi\partial^\mu\varphi + \lambda(u^\alpha\partial_\alpha\varphi)^2 + m^2\varphi^2], \tag{1}$$

where m is the mass of a scalar field; λ is the proportionality constant of Lorentz violation, usually a small quantity; u^α is a constant vector of an aether-like field in flat space-time; This article will use the sign difference of $(-,+,+,+)$.So the modified scalar field equation in flat space-time becomes

$$\partial_\mu\varphi\partial^\mu\varphi + \lambda(u^\alpha u^\beta\partial_\alpha\partial_\beta\varphi)^2 + m^2\varphi = 0. \tag{2}$$

In recent years, this modified scalar field equation has been deeply discussed, and it is found that scalar field equation in the field of high energy has many unique properties [28,29,30,31]. In the flat space-time of the regular coordinate system, the aether-like vector u^α is a constant and therefore naturally satisfies the relation

$$u^\alpha u_\beta = Const. \tag{3}$$

Now let's study the modification of the H-J equation for scalar particles in curved space-time. In curved space-time, in order to satisfy the relation of scalar field equation(2), u^α cannot be simply set as a constant, and the case of black hole with electric charge is taken into account, the action of the curved space-time is modified as follows

$$S = \int dx^4\sqrt{-g}\frac{1}{2}[(\partial_\mu\varphi - eA_\mu)(\partial^\mu\varphi - eA_\mu + \lambda(u^\alpha)\partial_\alpha\varphi)^2 + m^2\varphi^2]. \tag{4}$$

The Lorentz-violating scalar field equation in charged curve space-time can be obtained as

$$\begin{aligned}&\sqrt{-g}(\partial_\mu - eA_\mu)[g^{\mu\nu}\sqrt{-g}(\partial^\nu - eA_\nu)\varphi]\\&+\frac{\lambda}{\sqrt{-g}}(\partial_\mu - eA_\mu)[u^\mu u^\nu\sqrt{-g}(\partial_\nu - eA_\nu)\varphi] + m^2\varphi = 0,\end{aligned} \tag{5}$$

or expressed as

$$\frac{\lambda}{\sqrt{-g}}(\partial_\mu - eA_\mu)[\sqrt{-g}(g^{\mu\nu} + \lambda u^\mu u^\nu)(\partial_\nu - eA_\nu)\varphi] + m^2\varphi = 0. \tag{6}$$

This is the revised Klein-Gordon equation which means that the scalar field equation in curved space-time can be obtained by the gauge field transformation of $g^{\mu\nu} \to g^{\mu\nu} + \lambda u^\mu u^\nu$.In order to obtain the modified H-J equation in curved space-time, we can rewrite the scalar field wave function as

$$\varphi = \varphi C e^{\frac{i}{\hbar}S}. \tag{7}$$

Substitute equation (7) into any spin boson equation, treat $\hbar$ as a small quantity, and only keep the zero order term of the equation, and we can get

$$(g^{\mu\nu} + \lambda u^\mu u^\nu)(\partial_\mu - eA_\mu)(\partial_\nu - eA_\nu) + m^2 = 0. \tag{8}$$

This is the modified H-J equation for scalar particles with mass m, which will be used to study the Hawking tunneling radiation characteristics of black holes in section 3.

3 Correction on the Tunneling Radiation of a Global Monopole Charged Black Hole

The space-time line element of a global monopole charged black hole is[32]

$$\begin{aligned}ds^2 = &-(1 - 8\pi\eta^2 - \frac{2M}{r} + \frac{Q^2}{r^2})dt^2_{MC}\\&+(1 - 8\pi\eta^2 - \frac{2M}{r} + \frac{Q^2}{r^2})^{-1}dr^2 + r^2d\theta^2 + r^2sin^2\theta d\varphi^2,\end{aligned} \tag{9}$$

where dt_{MC} is the coordinate time of the black hole, η is a global monopole, M and Q are the mass and charge of the black hole respectively. Let

$$f(r) = 1 - 8\pi\eta^2 - \frac{2M}{r} + \frac{Q^2}{r^2}, \tag{10}$$

from the equation of the zero hypersurface, when $f(r) = 0$, we get

$$1 - 8\pi\eta^2 - \frac{2M}{r} + \frac{Q^2}{r^2} = 0. \tag{11}$$

By solving the equation (11), the event horizon r_h of the black hole can be expressed as

$$r_h = \frac{M + \sqrt{M^2 - (1 - 8\pi\eta^2)Q^2}}{1 - 8\pi\eta^2}. \tag{12}$$

The covariant metric tensor of the line element shown in equation (9) is

$$g_{\mu\nu} = \begin{bmatrix} g_{00} & 0 & 0 & 0 \\ 0 & g_{11} & 0 & 0 \\ 0 & 0 & g_{22} & 0 \\ 0 & 0 & 0 & g_{33} \end{bmatrix}, \tag{13}$$

$$\begin{aligned} g_{00} &= -f(r) \\ g_{11} &= f^{-1}(r) \\ g_{22} &= r^2 \\ g_{33} &= r^2 sin^2\theta \end{aligned}, \tag{14}$$

with whose metric determinant is

$$g = -r^4 sin^2\theta, \tag{15}$$

so the contravariant metric tensor can be calculated as

$$g^{\mu\nu} = \begin{bmatrix} g^{00} & 0 & 0 & 0 \\ 0 & g^{11} & 0 & 0 \\ 0 & 0 & g^{22} & 0 \\ 0 & 0 & 0 & g^{33} \end{bmatrix}, \tag{16}$$

$$\begin{aligned} g^{00} &= -\frac{1}{f(r)} \\ g^{11} &= f(r) \\ g^{22} &= \frac{1}{r^2} \\ g^{33} &= \frac{1}{r^2 sin^2\theta} \end{aligned}. \tag{17}$$

We're taking the four cases of $u^\alpha = \delta^\alpha_j$, $u_j = t, r, \theta, \varphi$ to discuss. Due to the action of particles S can be decomposed into

$$S = -\omega t + R(r) + Y(\theta, \varphi), \tag{18}$$

the equation (8) becomes

$$\begin{aligned} &[-\frac{1}{f(r)} + \lambda u^t u^t](-\omega - eA_t)^2 + [f(r) + \lambda u^r u^r][R^{'}(r) - eA_r]^2 \\ &+ (\frac{1}{r^2} + \lambda u^\theta u^\theta)(Y_{,\theta} - eA_\theta)^2 + (\frac{1}{r^2 sin^2\theta} + \lambda u^\varphi u^\varphi)(Y_{,\varphi} - eA_\varphi)^2 + m^2 = 0. \end{aligned} \tag{19}$$

Since this is a spherically symmetric black hole, in order to solve equation (19), we can only consider the modification in the case of $u^\alpha = \delta^\alpha_j u^j, j = t, r$.

According to equation (3), we can get $u^t = c_0/\sqrt{f(r)}$ and $u^r = c_1\sqrt{f(r)}$, where c_0 and c_1 are both constants, so we can rewrite the H-J equation (19) as

$$[-\frac{1}{f(r)}(1-\lambda c_0^2)](-\omega - eA_t)^2 + [f(r)(1+\lambda c_1^2)][R^{'}(r) - eA_r]^2 + \frac{C}{r^2} + m^2 = 0. \tag{20}$$

It follows that

$$R^{'}(r)_{\pm} = \pm\frac{\sqrt{(1-\lambda c_0^2)(-\omega - eA_t)^2 - f(r)(\frac{C}{r^2}+m^2)}}{f(r)\sqrt{1+\lambda c_1^2}} \pm eA_r. \tag{21}$$

According to the residue theorem, we can get

$$\begin{aligned} R(r)_{\pm} &= \pm\int_{r\to r_h}\frac{\sqrt{(1-\lambda c_0^2)(-\omega - eA_t)^2 - f(r)(\frac{C}{r^2}+m^2)}}{f(r)\sqrt{1+\lambda c_1^2}}dr \pm \int_{r\to r_h} eA_r dr \\ &= \pm i\pi\frac{(1-\lambda c_0^2/2)(1-\lambda c_1^2/2)(\omega + eA_t)}{f^{'}(r_h)} \pm eA_r r_h \\ &= \pm i\pi\frac{[1-\lambda(c_0^2+c_1^2)/2+\lambda^2 c_0^2 c_1^2/4](\omega + eA_t)}{f^{'}(r_h)} \pm eA_r r_h \end{aligned}. \tag{22}$$

The positive and negative signs here represent the outgoing and incoming solution respectively. The condition that λ is a small quantity has been used in the calculation.

The tunneling probability of the black hole is

$$\begin{aligned} \varGamma &= exp(-2I_m S) = exp[-2(I_m R_+ - I_m R_-)] \\ &= exp[-\frac{4\pi[1-\lambda(c_0^2+c_1^2)/2+\lambda^2 c_0^2 c_1^2/4](\omega + eA_t)}{f^{'}(r_h)}] \\ &= exp[-\frac{4\pi[1-\lambda(c_0^2+c_1^2)/2+\lambda^2 c_0^2 c_1^2/4](\omega - eQ/r_h)}{\frac{2M}{r_h^2}-\frac{2Q^2}{r_h^3}}] \\ &= exp(-\frac{\omega-\omega_0}{T_H}) \end{aligned}, \tag{23}$$

where T_H is the modified Hawking temperature

$$\begin{aligned} T_H &= [1-\lambda(c_0^2+c_1^2)/2+\lambda^2 c_0^2 c_1^2/4]\frac{1}{4\pi}(\frac{2M}{r_h^2}-\frac{2Q^2}{r_h^3}) \\ &= [1-\lambda(c_0^2+c_1^2)/2+\lambda^2 c_0^2 c_1^2/4]T_h \end{aligned}, \tag{24}$$

where

$$T_h = \frac{1}{4\pi}(\frac{2M}{r_h^2}-\frac{2Q^2}{r_h^3}). \tag{25}$$

It can be seen that when $\lambda > 0$, the modified Hawking temperature is higher than before the correction. Substitute the horizon surface equation (12) into equation (25), and get

$$T_h = \frac{M(1-8\pi\eta^2)^2[M+\sqrt{M^2-(1-8\pi\eta^2)Q^2}] - (1-8\pi\eta^2)^3 Q^2}{2\pi[M+\sqrt{M^2-(1-8\pi\eta^2)Q^2}]^3}. \tag{26}$$

T_h is the Hawking temperature of the black hole before correction. It can be seen from equation (24) that the modified Hawking temperature is related to the modified terms λ and λ^2. If $\lambda > 0$ is taken, then the modified Hawking temperature is higher than that before the correction. Another important physical quantity in the thermodynamic properties of black holes is the black hole entropy. According to the thermodynamics of black holes

$$dM = TdS + VdJ + UdQ, \tag{27}$$

where, V and U are respectively the spin potential and electromagnetic potential of the black hole. Therefore, the black hole entropy before correction at the event horizon r_h is

$$dS_h = \frac{dM - VdJ - UdQ}{T_h}. \tag{28}$$

The modified black hole entropy at event horizon r_h is

$$\begin{aligned} S_H &= \int dS_H = \int \frac{T_H dS_H}{T_H} = \int \frac{dM - VdJ - UdQ}{T_H} \\ &= \frac{dM - VdJ - UdQ}{[1 + \lambda(c_0^2 + c_1^2)/2 + \lambda^2 c_0^2 c_1^2/4]T_h} \\ &= [1 - \lambda(c_0^2 + c_1^2)/2 + \lambda^2 c_0^2 c_1^2/4] dS_h \\ &= S_h - S_h(\lambda) + S(\lambda^2) \end{aligned}. \tag{29}$$

It can be seen from the equations (28) and (29) that if $\lambda > 0$ and in the case of $u^\alpha = \delta_j^\alpha u^j, j = t, r$, the entropy of the black hole will be smaller than that before correction.

4 Conclusions

This paper has studied the Hawking tunneling radiation properties in the Lorentz invariance violating scalar field, and it is found that the Hawking radiation properties of black hole will be modified when $u^\alpha = \delta^\alpha$. If λ is a positive value, the Hawking temperature will be higher and the entropy of black hole will be smaller than before correction. The new result has been produced after the modification based on Lorentz invariance violation theory at high energy. This result is a supplement to the theory of the loss of information of black holes except mass, angular momentum and charge information. The lost information can be reflected in this little modified term. These results are very helpful for us to study theoretical physics and astrophysics.

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