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Memory Event-triggered Output Feedback Synchronization Control for Complex Dynamic Network with Bounded Distributed Delays

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Abstract In this paper, the method of memory event triggering output feedback is used to study the synchronization of complex dynamic network with bounded distributed delays when the target node is known or unknown. A memory event triggering scheme is proposed to reduce the transmission of data packets and shorten the transient process, and the network transmission delay is considered. The data packet signals released in recent times are stored at the sensor side and the controller side, which are used to generate event trigger function and design memory output feedback controller. By using Lyapunov stability theory, a sufficient condition for exponentially ultimately bounded of error dynamic system is given in the form of linear matrix inequalities (LMIs). Finally, an example proves the validity and feasibility of the theoretical results.

Keywords: complex dynamic network, memory event-triggered scheme, output feedback synchronization control, exponentially ultimately bounded, state estimation.

1 Introduction

Complex dynamic network is composed of huge interconnected power units. In recent decades, it has been widely concerned by people mainly because it exists in many natural systems or artificial systems. Social networks[1], biological networks[2], power grids[3], airport networks[4] and World Wide Web[5], which we often encounter but are not limited to, can be modeled and analyzed by complex dynamic networks. In recent years, people have made in-depth research on the dynamic behaviors of complex dynamic networks[6],[7],[8]. As one of the most studied, synchronization has made great progress. Including stochastic synchronization[9], anticipation synchronization[10], outer synchronization[11], and exponential synchronization[12], nonfragile exponential synchronization[13], globally exponential synchronization[7], etc.

As we all know, due to the limitation of signal transmission speed on complex dynamic network links and the problems of network hardware facilities, time delay is widespread. Time delay will reduce the performance of the system and affect the stability of the controlled system [14]. Therefore, it is of great significance to study the synchronization of complex dynamic networks with time delay, and has been widely studied. For example, the finite time lag synchronization of complex networks with coupled delays and master-slave complex networks with coupled time-varying delays is discussed respectively in[15],[16]. The finite-time hybrid projective synchronization of drive-response complex networks with distributed delays is studied in[17].

In complex dynamic networks, it is more reasonable to design discrete-time controllers because of the limited energy and computing resources of nodes. The traditional time-triggered sampling scheme has a sufficiently small sampling period to avoid the worst case. However, the time-triggered sampling scheme will produce many redundant sampling signals. Therefore, in order to avoid network-induced problems such as transmission delay and packet loss caused by data transmission and network bandwidth, event-triggered scheme is proposed and widely studied[18],[19],[20],[21],[22],[23]. In the event trigger scheme, the event generator will release the data packet signal only when the predetermined event trigger conditions are met. Generally speaking, the existing event-triggered schemes can be roughly divided into three categories: continuous event-triggered schemes[18],[19], discrete event-triggered schemes[20],[21],[22] and mixed event-triggered schemes[23]. In this paper, the discrete event-triggered scheme is studied, because it has the advantages of saving resources, reducing the computational burden of the controller and avoiding Zeno phenomenon.

It is worth noting that in the current event-triggered schemes, whether the sampled signal is released depends on two conditions. One is the threshold parameter, such as $\gamma(t)$ and θ in the event trigger conditions in[24],[25]. By adjusting the threshold, the conditions of event triggering can be changed to make it easier or less easy to trigger. The other is the error norm between the current sampled signal and the latest released signal[26],[27]. When the error norm is large, the current sampling signal can be released. It should be pointed out that these factors are not enough to truly reflect the dynamics of the system. For example, in the transient process, when the state is at peak or trough, the error norm between the current sampled signal and the latest released signal is very small, but the values of these two signals are much larger or smaller than those of other samples. In this case, we usually hope to release the current sampling signal to shorten the control time and realize synchronization. Therefore, the designed event trigger scheme should be related to several recently released data packet signals, that is, the memory event trigger control scheme[28],[29].

In this paper, the design of the controller will adopt memory events trigger output feedback synchronization control. Because in many control applications, complete state information is not available. Therefore, it is very important to study the controller based on output feedback, and output feedback is easier to implement than state feedback[30],[24]. In addition, the design of the controller will be related to the data packet signals released in recent times, so as to obtain better control performance.

Synchronization of complex dynamic networks with bounded distributed delays is studied in this paper when the target node is known or unknown. It is worth emphasizing that when the target node is unknown, we will first design an estimator to obtain the estimated state of the target node. Then, based on the estimated state, design the memory event triggers output feedback synchronous controller. At present, there have been many researches on the design of estimator[31],[32]. Finally, we summarized the main contributions as follows: (1) It is more practical to design memory event triggered feedback controllers when the target node is known or unknown. (2) The memory event trigger scheme is adopted, which reduces the conservative use of network bandwidth. Some newly released data packet signals are stored in the storage areas and used to design controller to improve system performance. (3) Sufficient conditions for synchronization of complex dynamic networks with bounded distributed delays are given.

The framework of this paper consists of six sections including this section. In the section 2, the model building, memory event trigger controller design and some necessary lemmas and assumptions are given. Synchronization analysis when the target node is known and unknown are given in the section 3 and the section 4 respectively. The section 5 gives an example of effectiveness. The conclusion will be presented in the section 6.

Notations: The notations used in this paper are quite standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ are n dimension Euclidean space and the set of $n \times m$ dimension real matrix, respectively. I_n is the $n \times n$ dimension identity matrix. A^T represents the transpose of matrix A, $\otimes$ stands for the Kronecker product of matrices. $\|\cdot\|$ is the Euclidean norm, and $|A| = trace\{A^T A\}$. $diag\{\cdots\}$ and $col\{\cdots\}$ represent diagonal matrix and column respectively.

2 Problem Formulation and Preliminaries

2.1 Complex Dynamical Network Modeling

In the time-delay complex dynamical network under consideration, the information of the ith node is as follows:

$$\begin{cases} \dot{x}_i(t) = f(x_i(t)) + \int_{t-\tau}^{t} \varphi(t-s)g(x_i(s))ds \ + \sum_{j=1}^{N} \text{ł}_{ij}\mathit{\Gamma} x_j(t) + u_i(t), \\ y_i(t) = C_i x_i(t), \qquad i = 1, 2, \cdots N, \end{cases} \tag{1}$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \cdots x_{in}(t)]^T$ is the state vector of the ith node, $y_i(t) = [y_{i1}(t), y_{i2}(t), \cdots y_{im}(t)]^T (1 \leq m \leq n)$ is the output information of the ith node. $C_i \in \mathbb{R}^{m\times n}$ is a constant matrix. $f(\cdot)$ and $g(\cdot)$ are non-linear vector-valued functions that satisfy certain conditions. τ is the bound of the distributed time delay, and $\varphi(\cdot)$ is the delay kernel function indicates the influence intensity of past history on the node dynamics. $L = [l_{ij}]_{N\times N}$ is the coupling configuration matrix, when node i can receive information from node j, $l_{ij} > 0$. Otherwise, $l_{ij} = 0$, and

$$l_{ii} = -\sum_{j=1, j\neq i}^{N} l_{ij}.$$

$\Gamma = diag\{\gamma_1, \gamma_2, \cdots \gamma_n\} > 0$ is the internal coupling matrix, $u_i(t)$ is the control input.

In this paper, our aim is to design memory event trigger controllers, which can synchronize the complex dynamical network with the target node when the target node is known or unknown.

Define $w(t) \in \mathbb{R}^n$ as the solution when the target node of complex dynamical network can be solved and satisfies:

$$\dot{w}(t) = f(w(t)) + \int_{t-\tau}^{t} \varphi(t-s) g(w(s)) ds. \tag{2}$$

Suppose the $w(t)$ is unique, the synchronization error $e_i(t) = x_i(t) - w(t)$ is defined, and the corresponding error dynamical system are

$$\dot{e}_i(t) = \tilde{f}(e_i(t)) + \int_{t-\tau}^{t} \varphi(t-s)\tilde{g}(e_i(s))ds + \sum_{j=1}^{N} \mathrm{l}_{ij} \Gamma e_j(t) + u_i(t), \qquad i = 1, 2, \cdots N, \tag{3}$$

where $\tilde{f}(e_i(t)) = f(x_i(t)) - f(w(t))$, $\tilde{g}(e_i(t)) = g(x_i(t)) - g(w(t))$.

2.2 Design of Memory Event Trigger Controller

In order to improve the system performance and make the complex dynamical network achieve synchronization as soon as possible, we use the memory event trigger controller instead of the traditional event trigger controller. Compared with the traditional event-triggered control, the memory event-triggered control has two storage areas at the sensor end and the controller end, which are used to store the recently released data packets. A detailed description of the memory event trigger control strategy flow can be seen in Fig.1.

At the sensor end, the sensor samples the output information of node i $(i = 1, 2, \cdots, N)$ with h as the sampling period, and writes it as $y_i(kh), (k = 1, 2, \cdots)$. Then, the memory event trigger function will determine whether to release the data packet. If the release condition is met, let $t_k^i = k$, store $y_i(t_k^i h)$ in the storage area and transfer it to the controller. For the controller end, $y_i(t_k^i h)$ will also be stored in the storage area, and the control input $u_i(t)$ will be generated by the stored information, and then $u_i(t)$ will be transmitted to the actuator. The zero-order holder can keep the signal continuous between two release moments.

For node i $(i = 1, 2, \cdots, N)$, the sampling sequence of the sensor is $\{0, h, 2h, \cdots\}$, and the data packets release moment sequence is $\{t_0^i h, t_1^i h, \cdots\}$. According to the above description, we can get $\{t_0^i, t_1^i \cdots\} \subset \{0, 1, 2, \cdots\}$ and $0 = t_0^i < t_1^i < t_2^i \cdots$. Different from the traditional event trigger, the memory event trigger uses not only the error between the current sampling moment and the last released data packet, but also the information of the previous release packets in the storage area. Assuming the last release instant is $t_k^i h$, and the next release instant $t_{k+1}^i h$ satisfies the following condition:

$$t_{k+1}^i h = t_k^i h + \min_{r\in\mathbb{N}}\{rh | \sum_{l=1}^{m} \varepsilon_l \delta_i^T(t_{k-l+1}) C_i^T C_i \delta_i(t_{k-l+1}) > \rho(t) \bar{e}_i^T(kh) C_i^T C_i \bar{e}_i(kh) + k_i\}, \tag{4}$$

where $\delta_i(t_{k-l+1}) = e_i(t_{k-l+1}^i h) - e_i(t_k^i h + rh)$, m is the storage area capacity, and $e_i(t_{k-l+1}^i h) = e_i(t_0^i h)$ if $k - l + 1 \leq 0$. ε_l is the weight parameter and satisfy $\sum_{l=1}^{m} \varepsilon_l = 1$. $\rho(t) = \rho_0 + \rho_1 e^{-\lambda \|e_i(t_k^i h + rh)\|^2}$, ρ_0, ρ_1, λ are given positive constants. It is easy to get $\rho_0 \leq \rho(t) \leq \rho_0 + \rho_1 \triangleq \rho$. $\bar{e}_i(kh) = \frac{1}{m}\sum_{l=1}^{m} e_i(t_{k-l+1} h)$, k_i is a known constant.

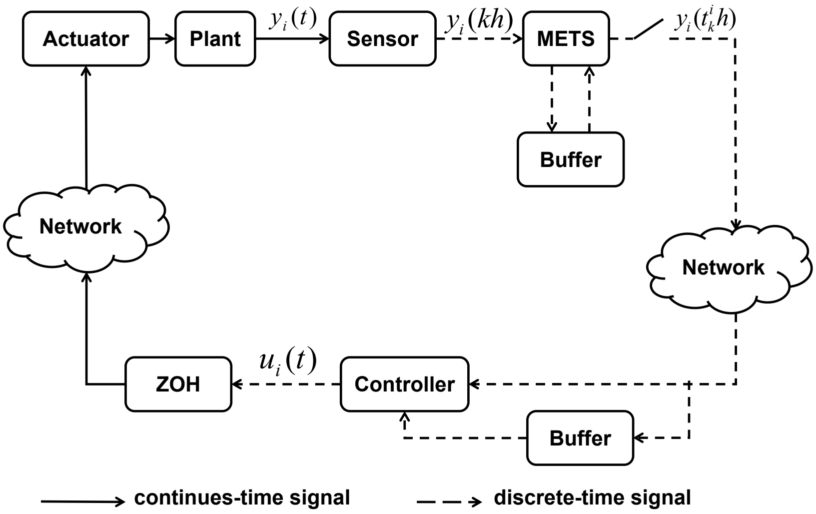

Figure 1. Framework of CDNs with METS.

Remark 1. *For the proposed memory event triggering strategy, the storage areas only store the data packet signals released by the recently* m *times* $\{y_i(t_k^i h), \cdots, y_i(t_{k-m+1}^i h)\}$. *It can be seen from the above that when* $m = 1$, *the memory event trigger control will become the traditional event trigger control.*

Remark 2. *The parameter* ε_l *is the weight parameter in the release condition. Generally, the release moment closer to the current moment has a larger weight, that is, the signals released at the latest moment is more important than before. So the* ε_1 *is larger than others and* $\varepsilon_i \geq \varepsilon_{i+1} (i = 2, 3, \cdots m-1)$. *It is easy to find that when* $\varepsilon_1 = 1, \varepsilon_i = 0 (i = 2, 3, \cdots m)$, *the memory event trigger control will also become the traditional event trigger control.*

Remark 3. *Visible from the* $\rho(t) = \rho_0 + \rho_1 e^{-\lambda\|e_i(t_k^i h + rh)\|^2}$, *the value of the* $\rho(t)$ *is related to the error norm at the current sampling time* $\lambda\|e_i(t_k^i h + rh)\|^2$. *The larger the* $\lambda\|e_i(t_k^i h + rh)\|^2$ *is and the smaller the* $\rho(t)$ *is, the easier it is to trigger. More release date packets will be delivered to the controller, thus improving the control performance.* λ *can control the rate of change,* ρ_1 *can adjust the proportion of the changed part in the equation* $\rho(t) = \rho_0 + \rho_1 e^{-\lambda\|e_i(t_k^i h + rh)\|^2}$, *so as to achieve the best control performance.*

To make the error dynamical system (3) exponentially ultimately bounded, the following memory feedback controller is designed:

$$u_i(t) = \sum_{l=1}^{m} \varepsilon_l K_l^i C_i e_i(t_{k-l+1}^i h), \qquad i = 1, 2, \cdots N, \tag{5}$$

where K_l^i is the controller gain to be designed.

In practice, when the network bandwidth is limited, but more information needs to be transmitted, or the transmission distance is long, the influence of network transmission delay should be considered. Assume that the network transmission delay of data packet $y_i(t_k^i h)$ is τ_k^i, $\tau_k^i \in [0, \tau_M^i]$, $\tau_M^i = \max_{k\in\mathbb{N}}\{\tau_k^i\}$. For $t \in [t_k^i h + \tau_k^i, t_{k+1}^i h + \tau_{k+1}^i)$, the control input remains unchanged due to the existence of ZOH.

Defining $\tau_i(t) = t - t_{k+1}^i h$, $t \in [t_k^i h + \tau_k^i, t_{k+1}^i h + \tau_{k+1}^i)$, then

$$u_i(t) = \sum_{l=1}^{m} \varepsilon_l K_l^i C_i(\delta_i(t_{k-l+1}) + e_i(t - \tau_i(t))). \tag{6}$$

Remark 4. *It can be seen from (5) that the designed memory feedback controller is related to the data packet signals released by node i for the last m times, which is the reason why the controller end needs storage area. Similarly, the general feedback controller can also be regarded as a memory feedback controller when* $m = \varepsilon_1 = 1$.

2.3 Assumption, Lemmas and Definition

Next, we will give some Assumption, Lemmas and Definition needed in this paper to complete the next part of the proof.

Assumption 1. *[33] For any* $x, z \in \mathbb{R}^n$*, the nonlinear vector-valued functions* f *and* g *are continuous and satisfy the following sector-bounded conditions:*

$$[f(x) - f(z) - U_1(x - z)]^T [f(x) - f(z) - U_2(x - z)] < 0,$$
$$[g(x) - g(z) - J_1(x - z)]^T [g(x) - g(z) - J_2(x - z)] < 0,$$

in which U_1*,* U_2*,* J_1*, and* J_2 *are real matrices of appropriate dimensions.*

Definition 1. *[34] (Exponentially Ultimately Bounded) The error dynamical system is exponentially ultimately bounded if there exist constants* $M > 0$*,* $\alpha > 0$ *and* $d > 0$ *such that*

$$\|e(t)\|^2 \leq M e^{-\alpha t} + d.$$

Lemma 1. *[35],[36] For given matrix* $S > 0$ *, if there exists real matrix* W *such that*

$$\begin{bmatrix} S & W^T \\ W & S \end{bmatrix} > 0,$$

then for function $\tau(t) \in (0, \tau_M]$*, and* $\dot{e}(t) : (0, \tau_M] \in \mathbb{R}^n$*, where* τ_M *is a positive constant, the following inequality holds:*

$$\tau_M \int_{t-\tau_M}^{t} \dot{e}^T(\theta) S \dot{e}(\theta) d\theta \geq \begin{bmatrix} e(t) \\ e(t - \tau(t)) \\ e(t - \tau_M) \end{bmatrix}^T \begin{bmatrix} S & * & * \\ W - S & 2S - 2W & * \\ -W & W - S & S \end{bmatrix} \begin{bmatrix} e(t) \\ e(t - \tau(t)) \\ e(t - \tau_M) \end{bmatrix}$$

.

Lemma 2. *[37] Let* M *be a positive semi-definite matrix,* $\alpha(\cdot) : (-\infty, a] \to [0, +\infty)$ *be a scalar function and* $\mathcal{F}(\cdot) : (-\infty, a] \to \mathbb{R}^n$ *be a vector function. If the integrations concerned are well defined, the following inequality holds:*

$$(\int_{-\infty}^{a} \alpha(s)\mathcal{F}(s)ds)^T M (\int_{-\infty}^{a} \alpha(s)\mathcal{F}(s)ds) \leq \int_{-\infty}^{a} \alpha(s)ds (\int_{-\infty}^{a} \alpha(s)\mathcal{F}^T(s) M \mathcal{F}(s)ds).$$

Lemma 3. *[38] (Schur Complement) For the given constant matrices* Σ_1*,* Σ_2*, and* Σ_3*, where* $\Sigma_1 = \Sigma_1^T$ *and* $\Sigma_2 > 0$*, then*

$$\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0,$$

if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0.$$

Lemma 4. *[39] For any vector* $x, y \in \mathbb{R}^n$*, and a positive definite matrix of appropriate dimension* Q*, the following inequality holds:*

$$2x^T y \leq x^T Q x + y^T Q^{-1} y.$$

3 Stability Analysis of Target Node Known

In this section, the exponentially ultimately bounded of error dynamical system (3) when the target node is known is given based on the proposed memory event triggering strategy (4). Detailed analysis is given in Theorem 1, and the control gain K_l will be solved in Theorem 2. For presentation convenience, in the following, we denote:

$$\begin{aligned} e(t-\tau(t)) &= [e_1^T(t-\tau_1(t)),\cdots,e_N^T(t-\tau_N(t))]^T, \\ F(e(t)) &= [\tilde{f}^T(e_1(t)),\tilde{f}^T(e_2(t)),\cdots,\tilde{f}^T(e_N(t))]^T, \\ G(e(t)) &= [\tilde{g}^T(e_1(t)),\tilde{g}^T(e_2(t)),\cdots,\tilde{g}^T(e_N(t))]^T, \\ \delta(t) &= [\delta_1^T(t),\delta_2^T(t),\cdots,\delta_N^T(t)]^T, \\ e(t) &= [e_1^T(t),e_2^T(t),\cdots,e_N^T(t)]^T, \\ \tau_M &= \max\{\tau_M^1,\tau_M^2,\cdots,\tau_M^N,\tau\}, \\ K_l &= diag\{K_l^1,K_l^2,\cdots,K_l^N\}, \end{aligned}$$

$$\begin{aligned} C &= diag\{C_1,C_2,\cdots,C_N\}, \\ P &= diag\{P_1,P_2,\cdots,P_N\}, \\ Q &= diag\{Q_1,Q_2,\cdots,Q_N\}, \\ R &= diag\{R_1,R_2,\cdots,R_N\}, \\ S &= diag\{S_1,S_2,\cdots,S_N\}, \\ W &= diag\{W_1,W_2,\cdots,W_N\}, \\ \Phi &= diag\{\Phi_1,\Phi_2,\cdots,\Phi_N\}, \\ \Psi &= diag\{\Psi_1,\Psi_2,\cdots,\Psi_N\}. \end{aligned}$$

Theorem 1. *Under Assumption 1, the error system (3) is exponentially ultimately bounded if there exist block diagonal matrices $P>0$, $Q>0$, $R>0$, $S>0$, diagonal matrices $\Phi>0$, $\Psi>0$ and matrix W, such that the following LMIs (7) and (8) hold for given parameters $\tau_M>0$ $\alpha>0$, $\rho>0$, $k_i>0 (i=1,2,\cdots,N)$, $\varepsilon_l\in[0,1] (l=1,2,\cdots,m)$ and the controller gain K_l $(l=1,2,\cdots,m)$. where*

$$\begin{bmatrix} S & W^T \\ W & S \end{bmatrix} > 0, \tag{7}$$

and

$$\begin{bmatrix} \Omega & H^T \\ H & -S^{-1} \end{bmatrix} < 0, \tag{8}$$

$$\begin{aligned} \Omega &= \begin{bmatrix} \Omega_{11} & * & * & * & * & * & * \\ \Omega_{21} & \Omega_{22} & * & * & * & * & * \\ e^{-\alpha\tau_M}W & \Omega_{32} & \Omega_{33} & * & * & * & * \\ \Omega_{41} & 0 & 0 & \Omega_{44} & * & * & * \\ \Omega_{51} & 0 & 0 & 0 & \Omega_{55} & * & * \\ P & 0 & 0 & 0 & 0 & \Omega_{66} & * \\ \Omega_{71} & \Omega_{72} & 0 & 0 & 0 & 0 & \Omega_{77} \end{bmatrix}, \\ \Omega_{11} &= \alpha P + P(L\otimes\Gamma) + (L\otimes\Gamma)^T P + R - e^{-\alpha\tau_M}S - (\Phi\otimes\breve{U}_1) - (\Psi\otimes\breve{J}_1), \\ \Omega_{21} &= \sum_{l=1}^{m}\varepsilon_l C^T K_l^T P - e^{-\alpha\tau_M}W + e^{-\alpha\tau_M}S,\ \Omega_{22} = \rho C^T C - 2e^{-\alpha\tau_M}S + 2e^{-\alpha\tau_M}W, \\ \Omega_{32} &= -e^{-\alpha\tau_M}W + e^{-\alpha\tau_M}S,\ \Omega_{33} = -e^{-\alpha\tau_M}R - e^{-\alpha\tau_M}S, \\ \Omega_{41} &= P + (\Phi\otimes\breve{U}_2)^T,\ \Omega_{44} = -(\Phi\otimes I),\ \Omega_{51} = (\Psi\otimes\breve{J}_2)^T, \\ \Omega_{55} &= \hat{\varphi}_\tau Q - (\Psi\otimes I),\ \Omega_{66} = -\frac{1}{\bar{\varphi}_\tau}Q,\ \Omega_{71} = col\{\varepsilon_1 C^T K_1^T P,\cdots,\varepsilon_m C^T K_m^T P\}, \\ \Omega_{72} &= col\{\frac{\rho}{m}C^T C,\cdots,\frac{\rho}{m}C^T C\},\ \Omega_{77} = \frac{\rho}{m^2}C^T C\times I_m + diag\{-\varepsilon_1 C^T C,\cdots,-\varepsilon_m C^T C\}, \\ \breve{U}_1 &= (U_1^T U_2 + U_2^T U_1)/2,\quad \breve{U}_2 = (U_1^T + U_2^T)/2,\quad \breve{J}_1 = (J_1^T J_2 + J_2^T J_1)/2,\quad \breve{J}_2 = (J_1^T + J_2^T)/2, \\ H &= \tau_M[(L\otimes\Gamma)\quad \sum_{l=1}^{m}\varepsilon_l K_l C\quad 0\quad I\quad 0\quad I\quad \varepsilon_1 K_1 C\cdots\varepsilon_m K_m C]. \end{aligned} \tag{9}$$

Proof. We introduce the following Lyapunov functional candidate:

$$V(t) := V_1(t) + V_2(t) + V_3(t) + V_4(t), \tag{10}$$

where

$$
\begin{aligned}
V_1(t) &= \sum_{i=1}^{N} e_i^T(t) P_i e_i(t),\\
V_2(t) &= \sum_{i=1}^{N} \int_0^\tau \varphi(\theta) e^{\alpha\theta} \int_{t-\theta}^{t} e^{\alpha(s-t)} \tilde{g}^T(e_i(s)) Q_i \tilde{g}(e_i(s)) ds d\theta,\\
V_3(t) &= \sum_{i=1}^{N} \int_{t-\tau_M}^{t} e^{\alpha(s-t)} e_i^T(s) R_i e_i(s) ds,\\
V_4(t) &= \sum_{i=1}^{N} \tau_M \int_{t-\tau_M}^{t} \int_{\theta}^{t} e^{\alpha(s-t)} \dot{e}_i^T(s) S_i \dot{e}_i(s) ds d\theta.
\end{aligned}
$$

According to (3), the time derivative of $V(t)$ is:

$$
\dot{V}(t) := \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t), \tag{11}
$$

where

$$
\begin{aligned}
\dot{V}_1(t) = {}& 2\sum_{i=1}^{N} e_i^T(t) P_i [\tilde{f}(e_i(t)) + \int_{t-\tau}^{t} \varphi(t-s) \tilde{g}(e_i(s)) ds \\
& + \sum_{j=1}^{N} \l_{ij} \Gamma e_j(t) + \sum_{l=1}^{m} \varepsilon_l K_l^i C_i(\delta_i(t_{k-l+1}) + e_i(t-\tau_i(t)))],\\
\dot{V}_2(t) = {}& -\alpha V_2(t) + \int_0^\tau \varphi(\theta) e^{\alpha\theta} d\theta \sum_{i=1}^{N} \tilde{g}^T(e_i(t)) Q_i \tilde{g}(e_i(t)) \\
& - \sum_{i=1}^{N} \int_0^\tau \varphi(\theta) e^{\alpha\theta} e^{-\alpha\theta} \tilde{g}^T(e_i(t-\theta)) Q_i \cdot \tilde{g}(e_i(t-\theta)) d\theta,\\
\dot{V}_3(t) = {}& -\alpha V_3(t) + \sum_{i=1}^{N} e_i^T(t) R_i e_i(t) - e^{-\alpha\tau_M} \sum_{i=1}^{N} e_i^T(t-\tau_M) \cdot R_i e_i(t-\tau_M),\\
\dot{V}_4(t) = {}& -\alpha V_4(t) - \sum_{i=1}^{N} \tau_M \int_{t-\tau_M}^{t} e^{\alpha(s-t)} \dot{e}_i^T(s) S_i \dot{e}_i(s) ds + \sum_{i=1}^{N} \tau_M^2 \dot{e}_i^T(t) S_i \dot{e}_i(t).
\end{aligned} \tag{12}
$$

It can be known from (4) that when the event trigger condition is not satisfied, one have:

$$
\rho(t) \bar{e}_i^T(kh) C_i^T C_i \bar{e}_i(kh) + k_i \geq \sum_{l=1}^{m} \varepsilon_l \delta_i^T(t_{k-l+1}) C_i^T C_i \delta_i(t_{k-l+1}). \tag{13}
$$

Let $\hat{\varphi}_\tau = \int_0^\tau \varphi(\theta) e^{\alpha\theta} d\theta$, $\bar{\varphi}_\tau = \int_0^\tau \varphi(s) ds$, by lemma 2, one can reach:

$$
\begin{aligned}
& -\int_0^\tau \varphi(\theta) \tilde{g}^T(e_i(t-\theta)) Q_i \tilde{g}(e_i(t-\theta)) d\theta \\
= & -\int_{t-\tau}^{t} \varphi(t-s) \tilde{g}^T(e_i(s)) Q_i \tilde{g}(e_i(s)) ds \\
\leq & -\frac{1}{\int_{t-\tau}^{t} \varphi(t-s) ds} [\int_{t-\tau}^{t} \varphi(t-s) \tilde{g}(e_i(s)) ds]^T \cdot Q_i \int_{t-\tau}^{t} \varphi(t-s) \tilde{g}(e_i(s)) ds \\
= & -\frac{1}{\int_0^\tau \varphi(s) ds} [\int_{t-\tau}^{t} \varphi(t-s) \tilde{g}(e_i(s)) ds]^T \cdot Q_i \int_{t-\tau}^{t} \varphi(t-s) \tilde{g}(e_i(s)) ds \\
= & -\frac{1}{\bar{\varphi}_\tau} [\int_{t-\tau}^{t} \varphi(t-s) \tilde{g}(e_i(s)) ds]^T \cdot Q_i \int_{t-\tau}^{t} \varphi(t-s) \tilde{g}(e_i(s)) ds.
\end{aligned} \tag{14}
$$

Furthermore, from Assumption 1, it can be obtained that:

$$\sum_{i=1}^{N} \Phi_i \begin{bmatrix} e_i(t) \\ \tilde{f}(e_i(t)) \end{bmatrix}^T \begin{bmatrix} \breve{U}_1 & -\breve{U}_2 \\ -\breve{U}_2^T & I \end{bmatrix} \begin{bmatrix} e_i(t) \\ \tilde{f}(e_i(t)) \end{bmatrix} \leq 0,$$

$$\sum_{i=1}^{N} \Psi_i \begin{bmatrix} e_i(t) \\ \tilde{g}(e_i(t)) \end{bmatrix}^T \begin{bmatrix} \breve{J}_1 & -\breve{J}_2 \\ -\breve{J}_2^T & I \end{bmatrix} \begin{bmatrix} e_i(t) \\ \tilde{g}(e_i(t)) \end{bmatrix} \leq 0,$$

which is equivalent to

$$\begin{aligned} & e^T(t)(\Phi \otimes \breve{U}_1)e(t) \ - 2e^T(t)(\Phi \otimes \breve{U}_2)F(e(t)) \ + F^T(e(t))(\Phi \otimes I)F(e(t)) \leq 0, \\ & e^T(t)(\Psi \otimes \breve{J}_1)e(t) \ - 2e^T(t)(\Psi \otimes \breve{J}_2)G(e(t)) \ + G^T(e(t))(\Psi \otimes I)G(e(t)) \leq 0. \end{aligned} \tag{15}$$

According to the above, the following can be obtained:

$$\begin{aligned} & \dot{V}(t) + \alpha V(t) \\ \leq & \ \alpha e^T(t)Pe(t) + 2e^T(t)P[F(e(t)) + \int_{t-\tau}^{t} \varphi(t-s) \cdot G(e(s))ds + (L \otimes \Gamma)e(t) \\ & + \sum_{l=1}^{m} \varepsilon_l K_l C(\delta(t_{k-l+1}) + e(t-\tau(t)))] + \hat{\varphi}_\tau G^T(e(t))QG(e(t)) \\ & - \frac{1}{\bar{\varphi}_\tau}[\int_{t-\tau}^{t} \varphi(t-s) \cdot G(e(s))ds]^T Q \int_{t-\tau}^{t} \varphi(t-s)G(e(s))ds + e^T(t)Re(t) \\ & - e^{-\alpha\tau_M} e^T(t-\tau_M)Re(t-\tau_M) - \tau_M \int_{t-\tau_M}^{t} e^{\alpha(s-t)} \dot{e}^T(s) \cdot S\dot{e}(s)ds + \tau_M^2 \dot{e}^T(t)S\dot{e}(t) \\ & + \rho(t)\bar{e}^T(kh)C^TC\bar{e}(kh) + \sum_{i=1}^{N} k_i - \sum_{l=1}^{m} \varepsilon_l \delta^T(t_{k-l+1})C^TC\delta(t_{k-l+1}) \\ & - e^T(t)(\Phi \otimes \breve{U}_1)e(t) + 2e^T(t)(\Phi \otimes \breve{U}_2)F(e(t)) - F^T(e(t))(\Phi \otimes I)F(e(t)) \\ & - e^T(t)(\Psi \otimes \breve{J}_1)e(t) + 2e^T(t)(\Psi \otimes \breve{J}_2)G(e(t)) - G^T(e(t))(\Psi \otimes I)G(e(t)). \end{aligned} \tag{16}$$

In the above formula, by using lemma 1, the following can be obtained:

$$\begin{aligned} -\tau_M \int_{t-\tau_M}^{t} e^{\alpha(s-t)} \dot{e}^T(s)S\dot{e}(s)ds \leq & \ -\tau_M e^{-\alpha\tau_M} \cdot \int_{t-\tau_M}^{t} \dot{e}^T(s)S\dot{e}(s)ds \\ \leq & \ - e^{-\alpha\tau_M} \cdot \begin{bmatrix} e(t) \\ e(t-\tau(t)) \\ e(t-\tau_M) \end{bmatrix}^T \begin{bmatrix} S & * & * \\ W-S & 2S-2W & * \\ -W & W-S & S \end{bmatrix} \begin{bmatrix} e(t) \\ e(t-\tau(t)) \\ e(t-\tau_M) \end{bmatrix}. \end{aligned} \tag{17}$$

Let

$$\begin{aligned} \xi(t) := [& e^T(t) \ e^T(t-\tau(t)) \ e^T(t-\tau_M) \ F^T(e(t)) \ G^T(e(t)) \\ & \int_{t-\tau}^{t} \varphi(t-s)G^T(e(s))ds) \quad \delta^T(t_k) \cdots \ \delta^T(t_{k-m+1})]^T. \end{aligned}$$

The following equation holds:

$$\tau_M^2 \dot{e}^T(t)S\dot{e}(t) = \xi^T(t)H^TSH\xi(t). \tag{18}$$

Therefore, by the definition of $\xi(t)$ and (11)-(18), there are

$$\dot{V}(t) + \alpha V(t) \leq \xi^T(t)\Omega\xi(t) + \xi^T(t)H^TSH\xi(t) + \sum_{i=1}^{N} k_i. \tag{19}$$

Combined inequality (8) with Schur Complement lemma, gives

$$\Omega + H^TSH < 0.$$

So there is

$$\dot{V}(t) + \alpha V(t) \leq \sum_{i=1}^{N} k_i. \tag{20}$$

According to the comparison principle, we can reach that:

$$V(t) \leq V(0)e^{-\alpha t} + \sum_{i=1}^{N} k_i \int_0^t e^{-\alpha(t-s)} ds. \tag{21}$$

It is easy to find $V(t) \geq \lambda_{min}(P)\|e(t)\|^2$, so we have:

$$\|e(t)\|^2 \leq \frac{V(0)}{\lambda_{min}(P)}e^{-\alpha t} + \frac{\sum_{i=1}^{N} k_i}{\alpha\lambda_{min}(P)}(1 - e^{-\alpha t}) \leq \frac{V(0)}{\lambda_{min}(P)}e^{-\alpha t} + \frac{\sum_{i=1}^{N} k_i}{\alpha\lambda_{min}(P)}. \tag{22}$$

At this point, the proof is complete. □

Theorem 2. *Under Assumption 1, the error system (3) is exponentially ultimately bounded if there exist block diagonal matrices $P > 0$, $Q > 0$, $R > 0$, $S > 0$, X_l, diagonal matrices $\Phi > 0$, $\Psi > 0$ and matrix W, such that the following LMIs (23) and (24) hold for given parameter $\tau_M > 0$, $\alpha > 0$, $\rho > 0$, $k_i > 0 (i = 1, 2, \cdots, N)$, $\varepsilon_l \in [0, 1] (l = 1, 2, \cdots, m)$. where*

$$\begin{bmatrix} S & W^T \\ W & S \end{bmatrix} > 0, \tag{23}$$

and

$$\begin{bmatrix} \bar{\Omega} & H^T P \\ PH & -2P + S \end{bmatrix} < 0, \tag{24}$$

$$\bar{\Omega} = \begin{bmatrix} \Omega_{11} & * & * & * & * & * & * \\ \bar{\Omega}_{21} & \Omega_{22} & * & * & * & * & * \\ e^{-\alpha\tau_M}W & \Omega_{32} & \Omega_{33} & * & * & * & * \\ \Omega_{41} & 0 & 0 & \Omega_{44} & * & * & * \\ \Omega_{51} & 0 & 0 & 0 & \Omega_{55} & * & * \\ P & 0 & 0 & 0 & 0 & \Omega_{66} & * \\ \bar{\Omega}_{71} & \Omega_{72} & 0 & 0 & 0 & 0 & \Omega_{77} \end{bmatrix},$$

$$\bar{\Omega}_{21} = \sum_{l=1}^{m} \varepsilon_l C^T X_l^T - e^{-\alpha\tau_M}W + e^{-\alpha\tau_M}S,$$

$$\bar{\Omega}_{71} = col\{\varepsilon_1 C^T X_1^T, \cdots, \varepsilon_m C^T X_m^T\}.$$

In addition, if LMIs (23) and (24) are solvable, the controller gain matrices are given as

$$K_l = P^{-1}X_l, \qquad l = 1, 2, \cdots, m.$$

Proof. For any constant ℓ, we can get $-PS^{-1}P \leq -2\ell P + \ell^2 S$, then LMI (24) can deduce

$$\begin{bmatrix} \bar{\Omega} & H^T P \\ PH & -PS^{-1}P \end{bmatrix} < 0. \tag{25}$$

Noting that $X_l = PK_l$, per-multiplying and post-multiplying (25) by $diag\{I, I, I, I, I, I, I, P^{-1}\}$, the LMI (8) can be obtained. The rest of the proof is the same as Theorem 1. □

4 Stability Analysis of Target Node Unknown

In the last section, we studied the controller design problem of synchronizing complex dynamic networks by using the memory event triggering strategy when the target node is known. And in this section, we will continue to study the controller design to synchronizing the complex dynamical network when the state information of the target node is unavailable. First, we will use the output information of the target node to estimate the state information, and then design a memory event trigger controller based on the state estimation to achieve synchronization.

4.1 The Design of Estimator

Assume that the target node information is as follows:

$$\begin{cases} \dot{w}(t) = f(w(t)) + \int_{t-\tau}^{t} \varphi(t-s)g(w(s))ds, \\ z(t) = Dw(t), \end{cases} \tag{26}$$

where $z(t)$ is the measurement output of the target node, $D \in \mathbb{R}^{m\times n}$ is a known constant matrix.

In the designed estimator, the sensor still uses the periodic sampling method. Assume that the sampling period is h_1, and the memory event trigger function will determine whether the sampled signal is transmitted to the estimator. The memory event trigger release sequence of the target node w is $0 = T_0h_1 < T_1h_1 < T_2h_1 < \cdots$, and it is determined by:

$$T_{k+1}h_1 = T_kh_1 + \min_{r\in\mathbb{N}}\{rh_1 | \hbar(\vartheta(T_{k-l+1}), \hat{\rho}(t), \hat{k}) > 0\}, \tag{27}$$

the memory event trigger function $\hbar(\vartheta(T_{k-l+1}), \hat{\rho}(t), \hat{k})$ will be given later, and it is related to the vector $\vartheta(T_{k-l+1})$, the given threshold constants $\hat{\rho}(t) > 0$, $\hat{k} > 0$.

Assume that the network transmission delay is η_k, where $\eta_k \in [0, \eta_M]$, $\eta_M = \max_{k\in\mathbb{N}}\{\eta_k, \tau\}$. For $t \in [T_kh_1 + \eta_k, T_{k+1}h_1 + \eta_{k+1})$, define $\eta(t) = t - T_{k+1}h_1$, and the estimate of the target node is given

$$\dot{\hat{w}}(t) = f(\hat{w}(t)) + \int_{t-\tau}^{t} \varphi(t-s)g(\hat{w}(s))ds + \sum_{l=1}^{m} \hat{\varepsilon}_l E_l D(w(T_{k-l+1}h_1) - \hat{w}(T_{k-l+1}h_1)), \tag{28}$$

where $\hat{w}(t)$ is the estimate of the state vector $w(t)$ and $E_l \in \mathbb{R}^{n\times m}$ is the estimator gain matrix, $\hat{\varepsilon}_l$ is the weight parameter.

The estimation error is defined as $\upsilon(t) = w(t) - \hat{w}(t)$, and the estimation error dynamics are as follows:

$$\dot{\upsilon}(t) = \breve{f}(\upsilon(t)) + \int_{t-\tau}^{t} \varphi(t-s)\breve{g}(\upsilon(s))ds - \sum_{l=1}^{m} \hat{\varepsilon}_l E_l D\upsilon(T_{k-l+1}h_1), \tag{29}$$

where $\breve{f}(\upsilon(t)) = f(w(t)) - f(\hat{w}(t)), \breve{g}(\upsilon(s)) = g(w(s)) - g(\hat{w}(s))$.

We define the event trigger function:

$$\hbar(\vartheta(T_{k-l+1}), \hat{\rho}(t), \hat{k}) := \sum_{l=1}^{m} \hat{\varepsilon}_l \vartheta^T(T_{k-l+1})D^TD\vartheta(T_{k-l+1}) - \hat{\rho}(t)\bar{\upsilon}^T(kh_1)D^TD\bar{\upsilon}(kh_1) - \hat{k}, \tag{30}$$

where $\vartheta(T_{k-l+1}) = \upsilon(T_{k-l+1}h_1) - \upsilon(T_kh_1 + rh_1)$ and $\bar{\upsilon}(kh_1) = \frac{1}{m}\sum_{l=1}^{m} \upsilon(T_{k-l+1}h_1)$, $\hat{\rho}(t) = \hat{\rho}_0 + \hat{\rho}_1 e^{-\hat{\lambda}\|\upsilon(T_kh_1+rh_1)\|^2}$, $\hat{\rho} = \hat{\rho}_0 + \hat{\rho}_1$.

For $t \in [T_kh_1 + \eta_k, T_{k+1}h_1 + \eta_{k+1})$, one can get:

$$\hat{\rho}(t)\bar{\upsilon}^T(kh_1)D^TD\bar{\upsilon}(kh_1) + \hat{k} \geq \sum_{l=1}^{m} \hat{\varepsilon}_l \vartheta^T(T_{k-l+1})D^TD\vartheta(T_{k-l+1}). \tag{31}$$

Remark 5. *For the designed estimator, the memory event trigger strategy is adopted here, which will achieve synchronization faster, and the obtained state estimation information will be closer to the real state information. As for the effectiveness of the estimator, that is, the estimation error dynamics are exponentially ultimately bounded, we will prove it by the following theorems.*

Theorem 3. *Under assumption 1, the estimation error system (29) is exponentially ultimately bounded if there exist matrices $\mathcal{P} > 0$, $\mathcal{Q} > 0$, $\mathcal{R} > 0$, $\mathcal{S} > 0$, real matrix $\mathcal{W}$ and two scalar constants $\phi > 0$, $\psi > 0$, such that the following LMIs (32) and (33) hold for given parameters $\eta_M > 0$, $\alpha_1 > 0$, $\hat{\rho} > 0$, $\hat{k} > 0$, $\hat{\varepsilon}_l \in [0, 1](l = 1, 2, \cdots, m)$ and the estimator gain E_l. where*

$$\begin{bmatrix} \mathcal{S} & \mathcal{W}^T \\ \mathcal{W} & \mathcal{S} \end{bmatrix} > 0, \tag{32}$$

and

$$\begin{bmatrix} \Delta & \mathcal{L}^T \\ \mathcal{L} & -\mathcal{S}^{-1} \end{bmatrix} < 0, \tag{33}$$

$$\Delta = \begin{bmatrix} \Delta_{11} & * & * & * & * & * & * \\ \Delta_{21} & \Delta_{22} & * & * & * & * & * \\ e^{-\alpha_1\eta_M}\mathcal{W} & \Delta_{32} & \Delta_{33} & * & * & * & * \\ \Delta_{41} & 0 & 0 & -\phi I & * & * & * \\ \psi \breve{J}_2^T & 0 & 0 & 0 & \Delta_{55} & * & * \\ \mathcal{P} & 0 & 0 & 0 & 0 & \Delta_{66} & * \\ \Delta_{71} & \Delta_{72} & 0 & 0 & 0 & 0 & \Delta_{77} \end{bmatrix},$$

$$\begin{aligned} \Delta_{11} =& \alpha_1\mathcal{P} + \mathcal{R} - e^{-\alpha_1\eta_M}\mathcal{S} - \phi\breve{U}_1 - \psi\breve{J}_1,\ \Delta_{21} = -\sum_{l=1}^{m}\hat{\varepsilon}_l D^T E_l^T \mathcal{P} - e^{-\alpha_1\eta_M}\mathcal{W} + e^{-\alpha_1\eta_M}\mathcal{S}, \\ \Delta_{41} =& \mathcal{P} + \phi\breve{U}_2^T,\ \Delta_{22} = \hat{\rho}D^T D - 2e^{-\alpha_1\eta_M}\mathcal{S} + 2e^{-\alpha_1\eta_M}\mathcal{W}, \\ \Delta_{71} =& col\{-\hat{\varepsilon}_1 D^T E_1^T\mathcal{P}, \cdots, -\hat{\varepsilon}_m D^T E_m^T \mathcal{P}\},\ \Delta_{32} = -e^{-\alpha_1\eta_M}\mathcal{W} + e^{-\alpha_1\eta_M}\mathcal{S}, \\ \Delta_{72} =& col\{\frac{\hat{\rho}}{m}D^T D, \cdots, \frac{\hat{\rho}}{m}D^T D\}\ \Delta_{33} = -e^{-\alpha_1\eta_M}\mathcal{R} - e^{-\alpha_1\eta_M}\mathcal{S},\ \Delta_{55} = \bar{\hat{\varphi}}_\tau\mathcal{Q} - \psi I, \\ \Delta_{66} =& -\frac{1}{\bar{\hat{\varphi}}_\tau}\mathcal{Q},\ \bar{\hat{\varphi}}_\tau = \int_0^\tau \varphi(\theta)e^{\alpha_1\theta}d\theta,\ \Delta_{77} = \frac{\hat{\rho}}{m^2}D^T D \times I_m + diag\{-\varepsilon_1 D^T D, \cdots, -\varepsilon_m D^T D\}, \\ \mathcal{L} =& \eta_M[0 \quad -\sum_{l=1}^{m}\hat{\varepsilon}_l E_l D \quad 0 \quad I \quad 0 \quad I \quad -\hat{\varepsilon}_1 E_1 D \quad \cdots - \hat{\varepsilon}_m E_m D]. \end{aligned} \tag{34}$$

Proof. The following Lyapunov candidate functions are given:

$$\begin{aligned} \mathcal{V}(t) =& \upsilon^T(t)\mathcal{P}\upsilon(t) + \int_0^\tau \varphi(\theta)e^{\alpha_1\theta}\int_{t-\theta}^{t} e^{\alpha_1(s-t)}\breve{g}^T(\upsilon(s))\mathcal{Q}\breve{g}(\upsilon(s))dsd\theta \\ &+ \int_{t-\eta_M}^{t} e^{\alpha_1(s-t)}\upsilon^T(s)\mathcal{R}\upsilon(s)ds + \eta_M\int_{t-\eta_M}^{t}\int_\theta^t e^{\alpha_1(s-t)}\dot{\upsilon}^T(s)\mathcal{S}\dot{\upsilon}(s)dsd\theta. \end{aligned} \tag{35}$$

Calculating the time derivative of $\mathcal{V}(t)$ along the trajectory of system (29), and then combine assumption 1, lemma 1,2 to get:

$$\begin{aligned} \dot{\mathcal{V}}(t) + \alpha_1\mathcal{V}(t) \leq& \alpha_1\upsilon^T(t)\mathcal{P}\upsilon(t) + 2\upsilon^T(t)\mathcal{P}[\breve{f}(\upsilon(t)) \\ &+ \int_{t-\tau}^{t}\varphi(t-s)\breve{g}(\upsilon(s))ds - \sum_{l=1}^{m}\hat{\varepsilon}_l E_l D(\vartheta(T_{k-l+1}) + \upsilon(t-\eta(t)))] \\ &+ \bar{\hat{\varphi}}_\tau\breve{g}^T(\upsilon(t))\mathcal{Q}\breve{g}(\upsilon(t)) - \frac{1}{\bar{\hat{\varphi}}_\tau}\cdot[\int_{t-\tau}^{t}\varphi(t-s)\breve{g}(\upsilon(s))ds]^T\mathcal{Q}[\int_{t-\tau}^{t}\varphi(t-s)\breve{g}(\upsilon(s))ds] \\ &+ \upsilon^T(t)\mathcal{R}\upsilon(t) - e^{-\alpha_1\eta_M}\upsilon^T(t-\eta_M)\mathcal{R}\upsilon(t-\eta_M) + \eta_M^2\dot{\upsilon}^T(t)\mathcal{S}\dot{\upsilon}(t) \\ &- e^{-\alpha_1\eta_M}\cdot\begin{bmatrix}\upsilon(t) \\ \upsilon(t-\eta(t)) \\ \upsilon(t-\eta_M)\end{bmatrix}^T\begin{bmatrix}\mathcal{S} & * & * \\ \mathcal{W}-\mathcal{S} & 2\mathcal{S}-2\mathcal{W} & * \\ -\mathcal{W} & \mathcal{W}-\mathcal{S} & \mathcal{S}\end{bmatrix}\begin{bmatrix}\upsilon(t) \\ \upsilon(t-\eta(t)) \\ \upsilon(t-\eta_M)\end{bmatrix} \\ &+ \hat{\rho}(t)\bar{\upsilon}^T(kh_1)D^T D\bar{\upsilon}(kh_1) + \hat{k} - \sum_{l=1}^{m}\hat{\varepsilon}_l\vartheta^T(T_{k-l+1})\cdot D^T D\vartheta(T_{k-l+1}) \\ &- \phi\upsilon^T(t)\breve{U}_1\upsilon(t) + 2\phi\upsilon^T(t)\breve{U}_2\breve{f}(\upsilon(t)) - \phi\breve{f}^T(\upsilon(t))I\breve{f}(\upsilon(t)) - \psi\upsilon^T(t)\breve{J}_1\upsilon(t) \\ &+ 2\psi\upsilon^T(t)\breve{J}_2\breve{g}(\upsilon(t)) - \psi\breve{g}^T(\upsilon(t))I\breve{g}(\upsilon(t)). \end{aligned} \tag{36}$$

Define vectors χ as follows:

$$\begin{aligned} \chi(t) := [&\upsilon^T(t) \quad \upsilon^T(t-\eta(t)) \quad \upsilon^T(t-\eta_M) \quad \breve{f}^T(\upsilon(t)) \quad \breve{g}^T(\upsilon(t)) \\ &\int_{t-\tau}^{t}\varphi(t-s)\breve{g}(\upsilon(s))ds \quad \vartheta^T(T_k)\cdots\vartheta^T(T_{k-l+1})]^T. \end{aligned}$$

Then (36) can be written as

$$\dot{\mathcal{V}}(t)+\alpha_1\mathcal{V}(t)\leq \chi^T(t)\varDelta\chi(t)+\chi^T(t)\mathcal{L}^T\mathcal{S}\mathcal{L}\chi(t)+\hat{k}. \tag{37}$$

Using lemma 3 to the inequality (33), we can get $\varDelta+\mathcal{L}^T\mathcal{S}\mathcal{L}<0$. Therefore, we have

$$\dot{\mathcal{V}}(t)+\alpha_1\mathcal{V}(t)\leq \hat{k}. \tag{38}$$

According to the comparison principle:

$$\mathcal{V}(t)\ \leq \mathcal{V}(0)e^{-\alpha_1 t}+\hat{k}\int_0^t e^{-\alpha_1(t-s)}ds, \tag{39}$$

and $\mathcal{V}(t)\geq \lambda_{min}(\mathcal{P})\|\upsilon(t)\|^2$, there are

$$\|\upsilon(t)\|^2\leq \frac{\mathcal{V}(0)}{\lambda_{min}(\mathcal{P})}e^{-\alpha_1 t}+\frac{\hat{k}}{\alpha_1\lambda_{min}(\mathcal{P})}. \tag{40}$$

end of proof. □

Theorem 4. *Under assumption 1, the estimation error system (29) is exponentially ultimately bounded if there exist matrices $\mathcal{P}>0$, $\mathcal{Q}>0$, $\mathcal{R}>0$, $\mathcal{S}>0$, Y_l, real matrix $\mathcal{W}$ and two scalar constants $\phi>0$, $\psi>0$, such that the following LMIs (41) and (42) hold for given parameters $\eta_M>0$, $\alpha_1>0$, $\hat{\rho}>0$, $\hat{k}>0$, $\hat{\varepsilon}_l\in[0,1](l=1,2,\cdots,m)$. where*

$$\begin{bmatrix}\mathcal{S} & \mathcal{W}^T\\ \mathcal{W} & \mathcal{S}\end{bmatrix}>0, \tag{41}$$

and

$$\begin{bmatrix}\bar{\varDelta} & \mathcal{L}^T\mathcal{P}\\ \mathcal{P}\mathcal{L} & -2\mathcal{P}+\mathcal{S}\end{bmatrix}<0, \tag{42}$$

$$\bar{\varDelta}=\begin{bmatrix}\varDelta_{11} & * & * & * & * & * & *\\ \bar{\varDelta}_{21} & \varDelta_{22} & * & * & * & * & *\\ e^{-\alpha_1\eta_M}\mathcal{W} & \varDelta_{32} & \varDelta_{33} & * & * & * & *\\ \varDelta_{41} & 0 & 0 & -\phi I & * & * & *\\ \psi\breve{J}_2^T & 0 & 0 & 0 & \varDelta_{55} & * & *\\ \mathcal{P} & 0 & 0 & 0 & 0 & \varDelta_{66} & *\\ \bar{\varDelta}_{71} & \varDelta_{72} & 0 & 0 & 0 & 0 & \varDelta_{77}\end{bmatrix},$$

$$\bar{\varDelta}_{21}=-\hat{\varepsilon}_l D^T Y_l^T-e^{-\alpha_1\eta_M}\mathcal{W}+e^{-\alpha_1\eta_M}\mathcal{S},$$
$$\bar{\varDelta}_{71}=col\{-\hat{\varepsilon}_1 D^T Y_1^T,\cdots,-\hat{\varepsilon}_m D^T Y_m^T\}.$$

Obviously, if LMIs (41) and (42) are solvable, the designed estimator gain matrix is given by

$$E_l=\mathcal{P}^{-1}Y_l,\quad l=1,2,\cdots,m.$$

4.2 Controller Design and Synchronic Analysis

Next, we will design a controller $\tilde{u}_i(t)$ to make the complex dynamic network achieve synchronization according to the estimated state of the target node w.

The synchronization error dynamical system can be described as:

$$\dot{e}_i(t)=\tilde{f}(e_i(t))+\int_{t-\tau}^t\varphi(t-s)\tilde{g}(e_i(s))ds+\sum_{j=1}^N l_{ij}\varGamma e_j(t)+\tilde{u}_i(t),\quad i=1,2,\cdots,N. \tag{43}$$

Here, the controller is still designed by the strategy of periodic sampling and memory event triggering. For the node i, it is assumed that the sampling period of the sensor is h_2. The event trigger sequence is $0 = t_0^i h_2 < t_1^i h_2 < t_2^i h_2 < \cdots$, and the release instant $t_{k+1}^i h_2$ is defined as follows:

$$t_{k+1}^i h_2 = t_k^i h_2 + \min_{r\in\mathbb{N}}\{rh_2 | \sum_{l=1}^{m} \tilde{\varepsilon}_l \hat{\delta}_i^T(t_{k-l+1}) C_i^T C_i \hat{\delta}_i(t_{k-l+1})) > \tilde{\rho}(t) \bar{\hat{e}}_i^T((kh_2) C_i^T C_i \bar{\hat{e}}_i(kh_2) + \tilde{k}_i\}, \tag{44}$$

where $\tilde{\varepsilon}_l$ is the weight parameter. $\hat{e}_i(t) = x_i(t) - \hat{w}(t)$, $\hat{\delta}_i(t_{k-l+1}) = \hat{e}_i(t_{k-l+1}^i h_2) - \hat{e}_i(t_k^i h_2 + rh_2)$, $\bar{\hat{e}}_i(kh_2) = \frac{1}{m}\sum_{l=1}^{m} \hat{e}_i(t_{k-l+1}^i h_2)$, $\tilde{\rho}(t) = \tilde{\rho}_0 + \tilde{\rho}_1 e^{-\tilde{\lambda}\|e(t_k^i h_2 + rh_2)\|^2}$, $\tilde{\rho} = \tilde{\rho}_0 + \tilde{\rho}_1$. $\tilde{k}_i$ is a known constant.

Design the controller as follows:

$$\tilde{u}_i(t) = \sum_{l=1}^{m} \tilde{\varepsilon}_l \tilde{K}_l^i C_i \hat{e}_i(t_{k-l+1}^i h_2), \tag{45}$$

where $\tilde{K}_l^i$ is the controller gain matrix to be designed. Consider the network transmission delay ζ_k^i, where $\zeta_k^i \in [0, \zeta_M^i]$, $\zeta_M^i = \max_{k\in\mathbb{N}}\{\zeta_k^i\}$. Let $\zeta_i(t) = t - t_{k+1}^i h_2$, $t \in [t_k^i h_2 + \zeta_k^i, t_{k+1}^i h_2 + \zeta_{k+1}^i)$.

The following symbols are given:

$$\begin{aligned} e(t-\zeta(t)) &= [e_1^T(t-\zeta_1(t)), \cdots, e_N^T(t-\zeta_N(t))]^T, \\ \hat{\delta}(t) &= [\hat{\delta}_1^T(t), \hat{\delta}_2^T(t), \cdots, \hat{\delta}_N^T(t)]^T, \\ \tilde{K}_l &= diag\{\tilde{K}_l^1, \tilde{K}_l^2, \cdots, \tilde{K}_l^N\}, \\ e(t_k h_2) &= [e_1^T(t_k^1 h_2), e_2^T(t_k^2 h_2), \cdots, e_N^T(t_k^N h_2)]^T, \\ \upsilon(t_k h_2) &= [\upsilon^T(t_k^1 h_2), \upsilon^T(t_k^2 h_2), \cdots, \upsilon^T(t_k^N h_2)]^T, \\ \zeta_M &= \max\{\zeta_M^1, \zeta_M^2, \cdots, \zeta_M^N, \tau\}, \end{aligned}$$

$$\begin{aligned} \tilde{P} &= diag\{\tilde{P}_1, \tilde{P}_2, \cdots, \tilde{P}_N\}, \\ \tilde{Q} &= diag\{\tilde{Q}_1, \tilde{Q}_2, \cdots, \tilde{Q}_N\}, \\ \tilde{R} &= diag\{\tilde{R}_1, \tilde{R}_2, \cdots, \tilde{R}_N\}, \\ \tilde{S} &= diag\{\tilde{S}_1, \tilde{S}_2, \cdots, \tilde{S}_N\}, \\ \tilde{W} &= diag\{\tilde{W}_1, \tilde{W}_2, \cdots, \tilde{W}_N\}, \\ \tilde{\Phi} &= diag\{\tilde{\Phi}_1, \tilde{\Phi}_2, \cdots, \tilde{\Phi}_N\}, \\ \tilde{\Psi} &= diag\{\tilde{\Psi}_1, \tilde{\Psi}_2, \cdots, \tilde{\Psi}_N\}. \end{aligned}$$

Theorem 5. *Under assumption 1, the error system (43) is exponentially ultimately bounded if there exist block diagonal matrices $\tilde{P} > 0$, $\tilde{Q} > 0$, $\tilde{R} > 0$, $\tilde{S} > 0$, diagonal matrices $\tilde{\Phi} > 0$, $\tilde{\Psi} > 0$ and real matrix $\tilde{W}$, such that the following LMIs (46) and (47) hold for given parameters $q_1 > 0$, $q_2 > 0$, $q_3 > 0$, $q_4 > 0$, $q_5 > 0$, $q_6 > 0$, $\zeta_M > 0$, $\alpha_2 > 0$, $\tilde{\rho} > 0$, $\tilde{k}_i > 0 (i = 1, 2, \cdots, N)$, $\beta \in (0, 1)$, $\tilde{\varepsilon}_l \in [0, 1](1, 2, \cdots, m)$ and the controller gain $\tilde{K}_l$. where*

$$\begin{bmatrix} \tilde{S} & \tilde{W}^T \\ \tilde{W} & \tilde{S} \end{bmatrix} > 0, \tag{46}$$

and

$$\begin{bmatrix} \Pi & \tilde{H}^T & \breve{H}^T \\ * & -(1+\frac{1}{q_6})^{-1}\tilde{S}^{-1} & 0 \\ * & * & \breve{\Pi} \end{bmatrix} < 0, \tag{47}$$

$$\Pi = \begin{bmatrix} \Pi_{11} & * & * & * & * & * & * \\ \Pi_{21} & \Pi_{22} & * & * & * & * & * \\ e^{-\alpha_2\zeta_M}\tilde{W} & \Pi_{32} & \Pi_{33} & * & * & * & * \\ 0 & 0 & 0 & \Pi_{44} & * & * & * \\ \Pi_{51} & 0 & 0 & 0 & \Pi_{55} & * & * \\ (\tilde{\Psi}\otimes\breve{J}_2)^T & 0 & 0 & 0 & 0 & \Pi_{66} & * \\ \tilde{P} & 0 & 0 & 0 & 0 & 0 & \Pi_{77} \end{bmatrix}, \tag{48}$$

$$
\begin{aligned}
\Pi_{11} =& \alpha_2 \tilde{P} + \tilde{P}(L\otimes \Gamma) + (L\otimes\Gamma)^T\tilde{P} + \tilde{R} - e^{-\alpha_2\zeta_M}\tilde{S} - (\tilde{\Phi}\otimes \breve{U}_1) - (\tilde{\Psi}\otimes\breve{J}_1),\\
\Pi_{21} =& -e^{-\alpha_2\zeta_M}\tilde{W} + e^{-\alpha_2\zeta_M}\tilde{S} + (1-\beta)\sum_{l=1}^{m}\tilde{\varepsilon}_l C^T\tilde{K}_l^T\tilde{P},\\
\Pi_{51} =& \tilde{P} + (\tilde{\Phi}\otimes\breve{U}_2)^T,\ \Pi_{22} = -2e^{-\alpha_2\zeta_M}\tilde{S} + 2e^{-\alpha_2\zeta_M}\tilde{W},\\
\Pi_{32} =& -e^{-\alpha_2\zeta_M}\tilde{W} + e^{-\alpha_2\zeta_M}\tilde{S},\ \Pi_{33} = -e^{-\alpha_2\zeta_M}\tilde{R} - e^{-\alpha_2\zeta_M}\tilde{S},\ \tilde{\hat{\varphi}}_\tau = \int_0^\tau \varphi(\theta)e^{\alpha_2\theta}d\theta,\\
\Pi_{44} =& diag\{\beta\tilde{\varepsilon}_1 C^T(q_1\otimes I)C,\cdots,\beta\tilde{\varepsilon}_m C^T(q_1\otimes I)C\} + (1-\beta)\frac{q_3\tilde{\rho}}{m^2}C^T((\frac{1}{q_4}+1)\otimes I)C\times I_m,\\
\Pi_{55} =& -(\tilde{\Phi}\otimes I),\ \Pi_{66} = \hat{\varphi}_\tau\tilde{Q} - (\tilde{\Psi}\otimes I)\ \Pi_{77} = -\frac{1}{\bar{\varphi}_\tau}\tilde{Q},\\
\tilde{H} =& \zeta_M[(L\otimes\Gamma)\quad 0\quad 0\quad \tilde{\varepsilon}_1\tilde{K}_1C\cdots\tilde{\varepsilon}_m\tilde{K}_mC\quad I\quad 0\quad I],\\
\breve{H} =& [\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_l^T\tilde{P}\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0],\\
\breve{\Pi} =& -\beta^{-1}((q_1+q_2)\otimes I) - (1-\beta)^{-1}((q_3+q_5)\otimes I).
\end{aligned}
$$

Proof. Consider the following Lyapunov function candidate:

$$
\begin{aligned}
\tilde{V}(t) =& \tilde{V}_1(t) + \tilde{V}_2(t) + \tilde{V}_3(t) + \tilde{V}_4(t),\\
\tilde{V}_1(t) =& \sum_{i=1}^{N} e_i^T(t)\tilde{P}_i e_i(t),\\
\tilde{V}_2(t) =& \sum_{i=1}^{N}\int_0^\tau \varphi(\theta)e^{\alpha_2\theta}\int_{t-\theta}^{t} e^{\alpha_2(s-t)}\tilde{g}^T(e_i(s))\tilde{Q}_i\tilde{g}(e_i(s))dsd\theta,\\
\tilde{V}_3(t) =& \sum_{i=1}^{N}\int_{t-\zeta_M}^{t} e^{\alpha_2(s-t)}e_i^T(s)\tilde{R}_i e_i(s)ds,\\
\tilde{V}_4(t) =& \sum_{i=1}^{N}\zeta_M\int_{t-\zeta_M}^{t}\int_\theta^t e^{\alpha_2(s-t)}\dot{e}_i^T(s)\tilde{S}_i\dot{e}_i(s)dsd\theta.
\end{aligned}
\tag{49}
$$

Combining Assumption 1, we can get the time derivative of $\tilde{V}(t)$

$$
\begin{aligned}
\dot{\tilde{V}}(t) + \alpha_2\tilde{V}(t) \le& \alpha_2 e^T(t)\tilde{P}e(t) + 2e^T(t)\tilde{P}[F(e(t)) + \int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds + (L\otimes\Gamma)e(t)]\\
&+ 2e^T(t)\tilde{P}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lC\hat{e}(t_{k-l+1}h_2) + \tilde{\hat{\varphi}}_\tau G^T(e(t))\tilde{Q}\cdot G(e(t))\\
&- \frac{1}{\bar{\varphi}_\tau}[\int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds]\tilde{Q}\cdot[\int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds] + e^T(t)\tilde{R}e(t)\\
&- e^{-\alpha_2\zeta_M}\cdot e^T(t-\zeta_M)\tilde{R}e(t-\zeta_M) - \zeta_M\int_{t-\zeta_M}^{t} e^{\alpha_2(s-t)}\dot{e}^T(s)\cdot\tilde{S}\dot{e}(s)ds + \zeta_M^2\dot{e}^T(t)\tilde{S}\dot{e}(t)\\
&- e^T(t)(\tilde{\Phi}\otimes\breve{U}_1)e(t) + 2e^T(t)(\tilde{\Phi}\otimes\breve{U}_2)F(e(t)) - F^T(e(t))(\tilde{\Phi}\otimes I)F(e(t))\\
&- e^T(t)(\tilde{\Psi}\otimes\breve{J}_1)e(t) + 2e^T(t)(\tilde{\Psi}\otimes\breve{J}_2)G(e(t)) - G^T(e(t))(\tilde{\Psi}\otimes I)G(e(t)),
\end{aligned}
\tag{50}
$$

where

$$
\begin{aligned}
&2e^T(t)\tilde{P}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lC\hat{e}(t_{k-l+1}h_2)\\
=&2\beta e^T(t)\tilde{P}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lC(e(t_{k-l+1}h_2)+\upsilon(t_{k-l+1}h_2))+2(1-\beta)e^T(t)\tilde{P}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lC(\hat{\delta}(t_{k-l+1})+\hat{e}(t-\zeta(t)))\\
=&2\beta e^T(t)\tilde{P}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lCe(t_{k-l+1}h_2)+2\beta e^T(t)\tilde{P}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lC\cdot\upsilon(t_{k-l+1}h_2)\\
&+2(1-\beta)e^T(t)\tilde{P}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lC\hat{\delta}(t_{k-l+1})+2(1-\beta)e^T(t)\tilde{P}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lC\hat{e}(t-\zeta(t))\\
\leq&\beta e^T(t)\tilde{P}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_l((q_1+q_2)\otimes I)^{-1}\tilde{K}_l^T\tilde{P}e(t)+\beta\sum_{l=1}^{m}\tilde{\varepsilon}_le^T(t_{k-l+1}h_2)C^T(q_1\otimes I)Ce(t_{k-l+1}h_2)\\
&+\beta\sum_{l=1}^{m}\tilde{\varepsilon}_l\upsilon^T(t_{k-l+1}h_2)C^T(q_2\otimes I)C\upsilon(t_{k-l+1}h_2)+(1-\beta)e^T(t)\tilde{P}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_l(q_3\otimes I)^{-1}\tilde{K}_l^T\tilde{P}e(t)\\
&+(1-\beta)q_3\sum_{l=1}^{m}\tilde{\varepsilon}_l\hat{\delta}^T(t_{k-l+1})C^TC\hat{\delta}(t_{k-l+1})+2(1-\beta)e^T(t)\tilde{P}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lC(e(t-\zeta(t))+\upsilon(t-\zeta(t))).
\end{aligned}
\tag{51}
$$

The above inequality can be obtained according to lemma 4.

In the above inequality, according to (44), when the event trigger condition is not satisfied, there are

$$
\begin{aligned}
&\sum_{l=1}^{m}\tilde{\varepsilon}_l\hat{\delta}^T(t_{k-l+1})C^TC\hat{\delta}(t_{k-l+1})\\
\leq&\frac{\tilde{\rho}}{m}\sum_{l=1}^{m}\hat{e}^T(t_{k-l+1}h_2)C^TC\frac{1}{m}\sum_{l=1}^{m}\hat{e}(t_{k-l+1}h_2)+\sum_{i=1}^{N}\tilde{k}_i\\
=&\frac{\tilde{\rho}}{m}\sum_{l=1}^{m}(e^T(t_{k-l+1}h_2)+\upsilon^T(t_{k-l+1}h_2))C^TC\frac{1}{m}\sum_{l=1}^{m}\cdot(e(t_{k-l+1}h_2)+\upsilon(t_{k-l+1}h_2))+\sum_{i=1}^{N}\tilde{k}_i\\
=&\frac{\tilde{\rho}}{m}\sum_{l=1}^{m}e^T(t_{k-l+1}h_2)C^TC\frac{1}{m}\sum_{l=1}^{m}e(t_{k-l+1}h_2)+\frac{2\tilde{\rho}}{m}\sum_{l=1}^{m}e^T(t_{k-l+1}h_2)C^TC\frac{1}{m}\sum_{l=1}^{m}\upsilon^T(t_{k-l+1}h_2)\\
&+\frac{\tilde{\rho}}{m}\sum_{l=1}^{m}\upsilon^T(t_{k-l+1}h_2)C^TC\frac{1}{m}\sum_{l=1}^{m}\upsilon^T(t_{k-l+1}h_2)+\sum_{i=1}^{N}\tilde{k}_i\\
\leq&\frac{\tilde{\rho}}{m}\sum_{l=1}^{m}e^T(t_{k-l+1}h_2)C^T((\frac{1}{q_4}+1)\otimes I)C\frac{1}{m}\cdot\sum_{l=1}^{m}e(t_{k-l+1}h_2)\\
&+\frac{\tilde{\rho}}{m}\sum_{l=1}^{m}\upsilon^T(t_{k-l+1}h_2)C^T\cdot((q_4+1)\otimes I)C\sum_{l=1}^{m}\upsilon(t_{k-l+1}h_2)+\sum_{i=1}^{N}\tilde{k}_i.
\end{aligned}
$$

Therefore, we can reach that

$$
\begin{aligned}
&(1-\beta)q_3\sum_{l=1}^{m}\tilde{\varepsilon}_l\hat{\delta}^T(t_{k-l+1})C^TC\hat{\delta}(t_{k-l+1})\\
\leq&(1-\beta)q_3[\frac{\tilde{\rho}}{m}\sum_{l=1}^{m}e^T(t_{k-l+1}h_2)C^T((\frac{1}{q_4}+1)\otimes I)C\cdot\frac{1}{m}\sum_{l=1}^{m}e(t_{k-l+1}h_2)\\
&+\frac{\tilde{\rho}}{m}\sum_{l=1}^{m}\upsilon^T(t_{k-l+1}h_2)C^T\cdot((q_4+1)\otimes I)C\frac{1}{m}\sum_{l=1}^{m}\upsilon(t_{k-l+1}h_2)+\sum_{i=1}^{N}\tilde{k}_i].
\end{aligned}
\tag{52}
$$

Similarly, we can get

$$
\begin{aligned}
&2(1-\beta)e^T(t)\tilde{P}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lC(e(t-\zeta(t))+\upsilon(t-\zeta(t))) \\
\le&2(1-\beta)e^T(t)\tilde{P}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lCe(t-\zeta(t))+(1-\beta)e^T(t)\tilde{P}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_l(q_5\otimes I)^{-1}\tilde{K}_l^T\tilde{P}e(t) \\
&+(1-\beta)\upsilon^T(t-\zeta(t))C(q_5\otimes I)C^T\upsilon(t-\zeta(t)).
\end{aligned}
\tag{53}
$$

For $\zeta_M^2\dot{e}^T(t)\tilde{S}\dot{e}(t)$, it can be written as follows

$$
\begin{aligned}
&\zeta_M^2\dot{e}^T(t)\tilde{S}\dot{e}(t) \\
=&\zeta_M^2[F(e(t))+\int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds+(L\otimes\Gamma)e(t)+\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lCe(t_{k-l+1}h_2) \\
&+\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lC\upsilon(t_{k-l+1}h_2)]^T\tilde{S}\cdot[F(e(t))+\int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds \\
&+(L\otimes\Gamma)e(t)+\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lCe(t_{k-l+1}h_2)+\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lC\upsilon(t_{k-l+1}h_2)] \\
=&\zeta_M^2[F(e(t))+\int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds+(L\otimes\Gamma)e(t)+\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lCe(t_{k-l+1}h_2)]^T \\
&\cdot\tilde{S}[F(e(t))+\int_{t-\tau}^{t}\varphi(t-s)\cdot G(e(s))ds+(L\otimes\Gamma)e(t)+\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lCe(t_{k-l+1}h_2)] \\
&\cdot+2\zeta_M^2[F(e(t))+\int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds+(L\otimes\Gamma)e(t)+\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lCe(t_{k-l+1}h_2)]^T \\
&\cdot\tilde{S}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lC\upsilon(t_{k-l+1}h_2)+\zeta_M^2\sum_{l=1}^{m}\tilde{\varepsilon}_l\upsilon^T(t_{k-l+1}h_2)C^T\tilde{K}_l^T\tilde{S}\tilde{K}_lC\sum_{l=1}^{m}\tilde{\varepsilon}_l\upsilon(t_{k-l+1}h_2).
\end{aligned}
$$

The block diagonal matrix $\tilde{S}$ can be written as $\tilde{S}=M^TM$, which is decomposed by Cholesky decomposition method, where M is a lower triangular real matrix with positive diagonal elements. Then the above formula can be written as

$$
\begin{aligned}
&2\zeta_M^2[F(e(t))+\int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds+(L\otimes\Gamma)e(t)+\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lCe(t_{k-l+1}h_2)]^T\tilde{S}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lC\upsilon(t_{k-l+1}h_2)\\
&=2\zeta_M^2[F(e(t))+\int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds+(L\otimes\Gamma)e(t)+\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lCe(t_{k-l+1}h_2)]^TM^TM\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lC\upsilon(t_{k-l+1}h_2)\\
&\leq\zeta_M^2\frac{1}{q_6}[F(e(t))+\int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds+(L\otimes\Gamma)e(t)+\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lCe(t_{k-l+1}h_2)]^TM^T\\
&\quad\cdot M[F(e(t))+\int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds+(L\otimes\Gamma)e(t)+\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lCe(t_{k-l+1}h_2)]\\
&\quad+\zeta_M^2q_6\cdot\sum_{l=1}^{m}\tilde{\varepsilon}_l\upsilon^T(t_{k-l+1}h_2)C^T\tilde{K}_l^TM^TM\tilde{K}_lC\sum_{l=1}^{m}\tilde{\varepsilon}_l\upsilon(t_{k-l+1}h_2)\\
&=\zeta_M^2\frac{1}{q_6}[F(e(t))+\int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds+(L\otimes\Gamma)e(t)+\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lCe(t_{k-l+1}h_2)]^T\\
&\quad\cdot\tilde{S}[F(e(t))+\int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds+(L\otimes\Gamma)e(t)+\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lCe(t_{k-l+1}h_2)]\\
&\quad+\zeta_M^2q_6\cdot\sum_{l=1}^{m}\tilde{\varepsilon}_l\upsilon^T(t_{k-l+1}h_2)C^T\tilde{K}_l^T\tilde{S}\tilde{K}_lC\sum_{l=1}^{m}\tilde{\varepsilon}_l\upsilon(t_{k-l+1}h_2).
\end{aligned}
$$

therefore

$$
\begin{aligned}
&\zeta_M^2\dot{e}^T(t)\tilde{S}\dot{e}(t)\\
=&\zeta_M^2(1+\frac{1}{q_6})[F(e(t))+\int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds+(L\otimes\Gamma)e(t)+\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lCe(t_{k-l+1}h_2)]^T\\
&\cdot\tilde{S}[F(e(t))+\int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds+(L\otimes\Gamma)e(t)+\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lC\cdot e(t_{k-l+1}h_2)]\\
&+\zeta_M^2(1+q_6)\sum_{l=1}^{m}\tilde{\varepsilon}_l\upsilon(t_{k-l+1}h_2)C^T\tilde{K}_l^T\cdot\tilde{S}\tilde{K}_lC\sum_{l=1}^{m}\tilde{\varepsilon}_l\upsilon(t_{k-l+1}h_2).
\end{aligned}
\tag{54}
$$

Then, the equality (50), combined with (51)-(54), and Lemma 1, the following can be obtained:

$$
\begin{aligned}
&\dot{\tilde{V}}(t)+\alpha_2\tilde{V}(t)\leq \alpha_2 e^T(t)\tilde{P}e(t)+2e^T(t)\tilde{P}[F(e(t))+\int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds+(L\otimes\Gamma)e(t)]\\
&\quad+\tilde{\bar{\varphi}}_\tau\cdot G^T(e(t))\tilde{Q}G(e(t))-\frac{1}{\bar{\varphi}_\tau}[\int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds]\tilde{Q}\cdot[\int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds]+e^T(t)\tilde{R}e(t)\\
&\quad-e^{-\alpha_2\zeta_M}\cdot e^T(t-\zeta_M)\tilde{R}e(t-\zeta_M)-e^{-\alpha_2\zeta_M}\cdot\begin{bmatrix}e(t)\\ e(t-\zeta(t))\\ e(t-\zeta_M)\end{bmatrix}^T\begin{bmatrix}\tilde{S} & * & *\\ \tilde{W}-\tilde{S} & 2\tilde{S}-2\tilde{W} & *\\ -\tilde{W} & \tilde{W}-\tilde{S} & \tilde{S}\end{bmatrix}\begin{bmatrix}e(t)\\ e(t-\zeta(t))\\ e(t-\zeta_M)\end{bmatrix}\\
&\quad-e^T(t)(\tilde{\Phi}\otimes\breve{U}_1)e(t)+2e^T(t)(\tilde{\Phi}\otimes\breve{U}_2)F(e(t))-F^T(e(t))(\tilde{\Phi}\otimes I)F(e(t))-e^T(t)(\tilde{\Psi}\otimes\breve{J}_1)e(t)\\
&\quad+2e^T(t)(\tilde{\Psi}\otimes\breve{J}_2)G(e(t))-G^T(e(t))(\tilde{\Psi}\otimes I)G(e(t))+\beta e^T(t)\tilde{P}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_l((q_1+q_2)\otimes I)^{-1}\tilde{K}_l^T\tilde{P}e(t)\\
&\quad+\beta\sum_{l=1}^{m}\tilde{\varepsilon}_l e^T(t_{k-l+1}h_2)C^T(q_1\otimes I)Ce(t_{k-l+1}h_2)+\beta\sum_{l=1}^{m}\tilde{\varepsilon}_l\upsilon^T(t_{k-l+1}h_2)C^T(q_2\otimes I)C\upsilon(t_{k-l+1}h_2)\\
&\quad+(1-\beta)e^T(t)\tilde{P}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_l((q_3+q_5)\otimes I)^{-1}\tilde{K}_l^T\tilde{P}e(t)\\
&\quad+(1-\beta)q_3[\frac{\tilde{\rho}}{m}\sum_{l=1}^{m}e^T(t_{k-l+1}h_2)C^T((\frac{1}{q_4}+1)\otimes I)C\cdot\frac{1}{m}\sum_{l=1}^{m}e(t_{k-l+1}h_2)\\
&\quad+\frac{\tilde{\rho}}{m}\sum_{l=1}^{m}\upsilon^T(t_{k-l+1}h_2)C^T((q_4+1)\otimes I)C\frac{1}{m}\sum_{l=1}^{m}\upsilon(t_{k-l+1}h_2)\\
&\quad+\sum_{i=1}^{N}\tilde{k}_i]+2(1-\beta)\cdot e^T(t)\tilde{P}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lCe(t-\zeta(t))+(1-\beta)\upsilon^T(t-\zeta(t))\cdot C^T(q_5\otimes I)C\upsilon(t-\zeta(t))\\
&\quad+\zeta_M^2(1+\frac{1}{q_6})[F(e(t))+\int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds+(L\otimes\Gamma)e(t)+\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lC\cdot e(t_{k-l+1}h_2)]^T\\
&\quad\cdot\tilde{S}[F(e(t))+\int_{t-\tau}^{t}\varphi(t-s)G(e(s))ds+(L\otimes\Gamma)e(t)+\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_lCe(t_{k-l+1}h_2)]\\
&\quad+\zeta_M^2(1+q_6)\cdot\sum_{l=1}^{m}\tilde{\varepsilon}_l\upsilon(t_{k-l+1}h_2)C^T\tilde{K}_l^T\tilde{S}\tilde{K}_lC\sum_{l=1}^{m}\tilde{\varepsilon}_l\upsilon(t_{k-l+1}h_2).
\end{aligned}
\tag{55}
$$

Letting

$$
\begin{aligned}
\tilde{\xi}(t):=[&e^T(t)\quad e^T(t-\zeta(t))\quad e^T(t-\zeta_M)\quad e^T(t_kh_2)\\
&\cdots e^T(t_{k-m+1}h_2)\quad F^T(e(t))\quad G^T(e(t))\quad \int_{t-\tau}^{t}\varphi(t-s)G^T(e(s))ds]^T.
\end{aligned}
$$

One can get

$$
\dot{\tilde{V}}(t)+\alpha_2\tilde{V}(t)\leq\ \tilde{\xi}^T(t)\tilde{\Pi}\tilde{\xi}(t)+(1+\frac{1}{q_6})\tilde{\xi}^T(t)\tilde{H}^T\tilde{S}\tilde{H}\tilde{\xi}(t)+(1-\beta)q_3\sum_{i=1}^{N}\tilde{k}_i+\Upsilon, \tag{56}
$$

where

$$
\begin{aligned}
\Upsilon=&(1-\beta)|C^T(q_5\otimes I)C|\|\upsilon(t-\zeta(t))\|^2+[\beta|C^T(q_2\otimes I)\cdot C|+(1-\beta)\frac{\tilde{\rho}}{m}|C^T((q_4+1)\otimes I)C|\\
&+\zeta_M^2(1+q_6)\cdot|\sum_{l=1}^{m}C^T\tilde{K}_l^T\tilde{S}\sum_{l=1}^{m}\tilde{K}_lC|]\|\upsilon(t_{k-l+1}h_2)\|^2,
\end{aligned}
$$

$$\begin{aligned}\|\upsilon(t-\zeta(t))\|^2 \leq& \frac{\mathcal{V}(0)}{\lambda_{min}(\mathcal{P})}e^{-\alpha_1(t-\zeta_M)} + \frac{\hat{k}}{\alpha_1\lambda_{min}(\mathcal{P})},\\ \|\upsilon(t_{k-l+1}h_2)\|^2 \leq& \frac{\mathcal{V}(0)}{\lambda_{min}(\mathcal{P})}e^{-\alpha_1(t_{k-m+1}h_2)} + \frac{\hat{k}}{\alpha_1\lambda_{min}(\mathcal{P})},\end{aligned}$$

$$\tilde{\Pi} = \begin{bmatrix} \Pi_0 & * & * & * & * & * & * \\ \Pi_{21} & \Pi_{22} & * & 0 & 0 & 0 & 0 \\ e^{-\alpha_2\zeta_M}\tilde{W} & \Pi_{32} & \Pi_{33} & 0 & 0 & 0 & 0 \\ 0 & * & * & \Pi_{44} & 0 & 0 & 0 \\ \Pi_{51} & * & * & * & \Pi_{55} & 0 & 0 \\ (\tilde{\Psi}\otimes\breve{J}_2)^T & * & * & * & * & \Pi_{66} & 0 \\ \tilde{P} & * & * & * & * & * & \Pi_{77} \end{bmatrix},$$

with

$$\Pi_0 = \Pi_{11} + \beta\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{P}\tilde{K}_l((q_1+q_2)\otimes I)^{-1}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_l^T\tilde{P} + (1-\beta)\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{P}\tilde{K}_l((q_3+q_5)\otimes I)^{-1}\sum_{l=1}^{m}\tilde{\varepsilon}_l\tilde{K}_l^T\tilde{P},$$

the definition of Π_{11} is the same as above. Apply Schur complement twice to (47) yields $\tilde{\Pi} + (1+\frac{1}{q_6})\tilde{H}^T\tilde{S}\tilde{H} < 0$, and

$$\|e(t)\|^2 \leq \frac{\tilde{V}(0)}{\lambda_{min}(\tilde{P})}e^{-\alpha_2 t} + \frac{(1-\beta)q_3\sum_{i=1}^{N}\tilde{k}_i + \Upsilon}{\alpha_2\lambda_{min}(\tilde{P})}, \tag{57}$$

so far, the proof is complete. □

Theorem 6. *Under assumption 1, the error system (43) is exponentially ultimately bounded if there exist block diagonal matrices $\tilde{P} > 0$, $\tilde{Q} > 0$, $\tilde{R} > 0$, $\tilde{S} > 0$, Z_l, diagonal matrices $\tilde{\Phi} > 0$, $\tilde{\Psi} > 0$ and real matrix $\tilde{W}$, such that the following LMIs (58) and (59) hold for given parameters $q_1 > 0$, $q_2 > 0$, $q_3 > 0$, $q_4 > 0$, $q_5 > 0$, $q_6 > 0$, $\zeta_M > 0$, $\alpha_2 > 0$, $\tilde{\rho} > 0$, $\tilde{k}_i > 0 (i = 1, 2, \cdots, N)$, $\beta \in (0, 1)$, $\tilde{\varepsilon}_l \in [0, 1] (l = 1, 2, \cdots, m)$ and the controller gain $\tilde{K}_l$. where*

$$\begin{bmatrix} \tilde{S} & \tilde{W}^T \\ \tilde{W} & \tilde{S} \end{bmatrix} > 0, \tag{58}$$

and

$$\begin{bmatrix} \bar{\Pi} & \tilde{H}^T\tilde{P} & \bar{H}^T \\ * & (-2\tilde{P}+\tilde{S})(1+\frac{1}{q_6})^{-1} & 0 \\ * & * & \breve{\Pi} \end{bmatrix} < 0, \tag{59}$$

where

$$\bar{\Pi} = \begin{bmatrix} \Pi_{11} & * & * & * & * & * & * \\ \bar{\Pi}_{21} & \Pi_{22} & * & * & * & * & * \\ e^{-\alpha_2\zeta_M}\tilde{W} & \Pi_{32} & \Pi_{33} & * & * & * & * \\ 0 & 0 & 0 & \Pi_{44} & * & * & * \\ \Pi_{51} & 0 & 0 & 0 & \Pi_{55} & * & * \\ (\tilde{\Psi}\otimes\breve{J}_2)^T & 0 & 0 & 0 & 0 & \Pi_{66} & * \\ \tilde{P} & 0 & 0 & 0 & 0 & 0 & \Pi_{77} \end{bmatrix},$$

$$\bar{\Pi}_{21} = -e^{-\tilde{\alpha}\zeta_M}\tilde{W} + e^{-\tilde{\alpha}\zeta_M}\tilde{S} + (1-\beta)\sum_{l=1}^{m}\tilde{\varepsilon}_l C^T Z_l^T, \quad \bar{H} = [Z_l^T \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \tag{60}$$

other definitions are the same as Theorem 5. Moreover, if the LMIs (58) and (59) are solvable, the desired controller gain matrices are given as

$$\tilde{K}_l = \tilde{P}^{-1}Z_l, \qquad l = 1, 2, \cdots, m.$$

5 Numerical Example

In the previous sections, the sufficient conditions for the synchronization of complex dynamic networks with bounded distributed delays are given by the method of memory event triggering. In this section, an example is given to verify the validity and feasibility of the theoretical results. Let the nonlinear function $f(x_i(t))$ to be Chua's circuit[40]. Then $f(x_i(t))$ can be described as follows:

$$f(x_i(t)) = \begin{pmatrix} a(-x_{i1} + x_{i2} - o(x_{i1})) \\ x_{i1} - x_{i2} + x_{i3} \\ -bx_{i2} \end{pmatrix}, \tag{61}$$

where $o(x_{i1}) = dx_{i1} + \frac{1}{2}(c-d)(|x_{i1}(t)+1| - |x_{i1}(t)-1|)$, $a = 10$, $b = 18$, $c = -\frac{4}{3}$, $d = -\frac{3}{4}$.

Let

$$g(x_i(t)) = \begin{pmatrix} -0.8x_{i1} + 0.2(|x_{i1}(t)+8| - |x_{i1}(t)-8|) + 0.4x_{i2} \\ 0.8x_{i2} + \tanh(-0.6x_{i2}) \\ 0.6x_{i3} + \tanh(-0.4x_{i3}) \end{pmatrix}, \tag{62}$$

$\varphi(t) = e^{-t}$, $\tau = 2$. Let

$$U_1 = \begin{bmatrix} -ad-a & a & 0 \\ 1 & -1.2 & 1 \\ 0 & -a & -1.8 \end{bmatrix},\ U_2 = \begin{bmatrix} -ad-a & a & 0 \\ 1 & 0.8 & 1 \\ 0 & -a0.8 \end{bmatrix},\ J_1 = \begin{bmatrix} -0.8 & 0.4 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.6 \end{bmatrix},\ J_2 = \begin{bmatrix} -0.4 & 0.4 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix},$$

which is easy to verify, the sector-bounded condition of nonlinear vector-valued functions in Assumption 1 is satisfied.

In the event trigger condition, let $\rho_0 = 0.2$, $\rho_1 = 0.2$, $\lambda = 0.1$, $\rho(t) \leq \rho_0 + \rho_1 = 0.4$, $k_i = 0.3 (i = 1, 2, 3)$. The memory area capacity $m = 2$, $\varepsilon_1 = 0.7$, $\varepsilon_2 = 0.3$. In addition, $\tau_M = 2$,

$$A = \begin{bmatrix} -5 & 3 & 2 \\ 3 & -4 & 1 \\ 2 & 1 & -3 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix},$$

and $\Gamma = 3I$. When the target node is known, a feasible solution is obtained by solving inequality (7), (8) by using Matlab LMIs Toolbox:

$$P_1 = \begin{bmatrix} 0.0297 & -0.0007 & -0.0021 \\ * & 0.0309 & 0.0002 \\ * & * & 0.0300 \end{bmatrix},\ P_2 = \begin{bmatrix} 0.0378 & -0.0013 & -0.0025 \\ * & 0.0390 & 0.0002 \\ * & * & 0.0379 \end{bmatrix},\ P_3 = \begin{bmatrix} 0.0596 & -0.0024 & -0.0031 \\ * & 0.0608 & 0.0005 \\ * & * & 0.0599 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 1.7096 & -0.4118 & 0.0293 \\ * & 2.1723 & 0.0088 \\ * & * & 2.4380 \end{bmatrix},\ Q_2 = \begin{bmatrix} 1.7304 & -0.4118 & 0.0301 \\ * & 2.1928 & 0.0088 \\ * & * & 2.4594 \end{bmatrix},\ Q_3 = \begin{bmatrix} 1.7449 & -0.4114 & 0.0308 \\ * & 2.2076 & 0.0086 \\ * & * & 2.4739 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 0.4797 & -0.3636 & -0.0307 \\ * & 0.2871 & 0.0523 \\ * & * & 0.3509 \end{bmatrix},\ R_2 = \begin{bmatrix} 0.5222 & -0.3697 & -0.0330 \\ * & 0.3260 & 0.0537 \\ * & * & 0.3948 \end{bmatrix},\ R_3 = \begin{bmatrix} 0.5471 & -0.3727 & -0.0360 \\ * & 0.3507 & 0.0547 \\ * & * & 0.4206 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 0.9918 & 0.0175 & 0.0282 \\ * & 0.9765 & -0.0055 \\ * & * & 0.9942 \end{bmatrix},\ S_2 = \begin{bmatrix} 0.9949 & 0.0177 & 0.0283 \\ * & 0.9795 & -0.0054 \\ * & * & 0.9973 \end{bmatrix},\ S_3 = \begin{bmatrix} 1.0018 & 0.0166 & 0.0284 \\ * & 0.9855 & -0.0052 \\ * & * & 1.0036 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} -0.2849 & 0.0541 & -0.1800 \\ * & -0.0700 & 0.0023 \\ * & * & -0.2675 \end{bmatrix},\ W_2 = \begin{bmatrix} -0.2901 & 0.0551 & -0.1800 \\ * & -0.0757 & 0.0015 \\ * & * & -0.2731 \end{bmatrix},\ W_3 = \begin{bmatrix} -0.2908 & 0.0548 & -0.1795 \\ * & -0.0768 & 0.0017 \\ * & * & -0.2742 \end{bmatrix},$$

$$\varPhi = diag\{0.0199 \quad 0.0203 \quad 0.0203\}, \quad \varPsi = diag\{5.6412 \quad 5.6783 \quad 5.6963\},$$

$$K_1^1 = \begin{bmatrix} -0.6394 \\ -0.0666 \\ -0.6486 \end{bmatrix}, K_1^2 = \begin{bmatrix} -0.4969 \\ -0.0589 \\ -0.5067 \end{bmatrix}, K_1^3 = \begin{bmatrix} -0.3086 \\ 0.0388 \\ -0.3145 \end{bmatrix},$$

$$K_2^1 = \begin{bmatrix} 0.1298 \\ -0.0303 \\ 0.1583 \end{bmatrix}, K_2^2 = \begin{bmatrix} 0.1092 \\ -0.0447 \\ 0.1219 \end{bmatrix}, K_2^3 = \begin{bmatrix} 0.2219 \\ -0.0208 \\ 0.2338 \end{bmatrix}.$$

By solving Theorem 1, the complex dynamic network can achieve synchronization for given parameters.

When the target node is unknown, the estimator is designed first. Suppose $\eta_M = 3$, $\tau = 2$, $\alpha_1 = 0.3$, $\hat{\rho} = 0.4$ and $\hat{\varepsilon_1} = 0.6$, $\hat{\varepsilon}_2 = 0.4$. By solving inequality (32), (33), we can get:

$$\mathcal{P} = \begin{bmatrix} 0.0424 & -0.0055 & 0.0022 \\ * & 0.0440 & -0.0001 \\ * & * & 0.0476 \end{bmatrix}, \ \mathcal{Q} = \begin{bmatrix} 0.6355 & -0.2906 & 0.2102 \\ * & 1.0216 & -0.0942 \\ * & * & 1.1824 \end{bmatrix}, \ \mathcal{R} = \begin{bmatrix} -0.1110 & -0.1084 & -0.0660 \\ * & -0.1622 & 0.0090 \\ * & * & -0.1658 \end{bmatrix},$$

$$\mathcal{S} = \begin{bmatrix} 0.4329 & 0.0009 & 0.0444 \\ * & 0.3994 & -0.0043 \\ * & * & 0.4381 \end{bmatrix}, \ \mathcal{W} = \begin{bmatrix} -0.4370 & 0.0278 & -0.3338 \\ * & -0.0734 & 0.0075 \\ * & * & -0.4268 \end{bmatrix}, \ E_1 = \begin{bmatrix} -0.3097 \\ -0.0213 \\ -0.2535 \end{bmatrix}, \ E_2 = \begin{bmatrix} 0.6740 \\ 0.1246 \\ 0.5910 \end{bmatrix},$$

and $\phi = 0.0046$, $\psi = 5.1336$.

Let $\zeta_M = 2$, $\tau = 2$, $\alpha_2 = 0.2$, $\tilde{\varepsilon}_1 = 0.6$, $\tilde{\varepsilon}_2 = 0.4$, $q_1 = q_2 = 0.5$, $q_3 = q_4 = 0.4$, $q_5 = q_6 = 0.8$, $\beta = 0.4$, $\tilde{\rho} = 0.4$. Combined with the design of the estimator solve inequality (46), (47), we can get:

$$\tilde{P}_1 = \begin{bmatrix} 0.0050 & -0.0002 & 0.0000 \\ * & 0.0047 & 0.0000 \\ * & * & 0.0049 \end{bmatrix}, \ \tilde{P}_2 = \begin{bmatrix} 0.0061 & -0.0003 & 0.0000 \\ * & 0.0058 & 0.0000 \\ * & * & 0.0060 \end{bmatrix}, \ \tilde{P}_3 = \begin{bmatrix} 0.0091 & -0.0005 & 0.0000 \\ * & 0.0086 & 0.0001 \\ * & * & 0.0090 \end{bmatrix},$$

$$\tilde{Q}_1 = \begin{bmatrix} 0.6161 & -0.2509 & 0.0001 \\ * & 0.8305 & 0.0114 \\ * & * & 1.0386 \end{bmatrix}, \ \tilde{Q}_2 = \begin{bmatrix} 0.6212 & -0.2517 & 0.0001 \\ * & 0.8360 & 0.0116 \\ * & * & 1.0453 \end{bmatrix}, \ \tilde{Q}_3 = \begin{bmatrix} 0.6243 & -0.2522 & 0.0001 \\ * & 0.8395 & 0.0116 \\ * & * & 1.0494 \end{bmatrix},$$

$$\tilde{R}_1 = \begin{bmatrix} 0.0777 & -0.0873 & 0.0034 \\ * & -0.0210 & 0.0112 \\ * & * & 0.0433 \end{bmatrix}, \ \tilde{R}_2 = \begin{bmatrix} 0.0830 & -0.0882 & 0.0035 \\ * & -0.0170 & 0.0115 \\ * & * & 0.0487 \end{bmatrix}, \ \tilde{R}_3 = \begin{bmatrix} 0.0859 & -0.0886 & 0.0035 \\ * & -0.0148 & 0.0116 \\ * & * & 0.0517 \end{bmatrix},$$

$$\tilde{S}_1 = \begin{bmatrix} 0.6660 & 0.0068 & -0.0006 \\ * & 0.6737 & -0.0010 \\ * & * & 0.6676 \end{bmatrix}, \ \tilde{S}_2 = \begin{bmatrix} 0.6666 & 0.0069 & -0.0006 \\ * & 0.6744 & -0.0010 \\ * & * & 0.6682 \end{bmatrix}, \ \tilde{S}_3 = \begin{bmatrix} 0.6669 & 0.0068 & -0.0006 \\ * & 0.6745 & -0.0009 \\ * & * & 0.6683 \end{bmatrix},$$

$$\tilde{W}_1 = \begin{bmatrix} -0.0026 & 0.0212 & -0.0014 \\ * & 0.0219 & -0.0029 \\ * & * & 0.0037 \end{bmatrix}, \ \tilde{W}_2 = \begin{bmatrix} -0.0036 & 0.0213 & -0.0014 \\ * & 0.0210 & -0.0030 \\ * & * & 0.0026 \end{bmatrix}, \ \tilde{W}_3 = \begin{bmatrix} -0.0042 & 0.0213 & -0.0014 \\ * & 0.0205 & -0.0030 \\ * & * & 0.0020 \end{bmatrix},$$

$$\varPhi = diag\{0.0047 \quad 0.0047 \quad 0.0048\}, \quad \varPsi = diag\{3.4222 \quad 3.4360 \quad 3.4440\},$$

$$K_1^1 = \begin{bmatrix} -0.1075 \\ -0.0160 \\ -0.1080 \end{bmatrix}, K_1^2 = \begin{bmatrix} -0.0869 \\ -0.0135 \\ -0.0870 \end{bmatrix}, K_1^3 = \begin{bmatrix} -0.0579 \\ -0.0091 \\ -0.0576 \end{bmatrix},$$

$$K_2^1 = \begin{bmatrix} -1.5298 \\ -0.2910 \\ -1.5809 \end{bmatrix}, K_2^2 = \begin{bmatrix} -1.2402 \\ -0.2453 \\ -1.2793 \end{bmatrix}, K_2^3 = \begin{bmatrix} -0.8284 \\ -0.1665 \\ -0.8508 \end{bmatrix}.$$

It can be proved that when the target node is unknown, the memory output feedback controller designed based on the estimated state of the target node can synchronize the complex dynamic network.

6 Conclusion

This paper discusses the design of synchronous controllers for complex dynamic networks with bounded distributed delays when the target node is known or unknown. The memory event trigger control strategy is adopted. Compared with the traditional event-triggered control method, it has the advantages of reducing the transmission frequency of data packet signals, shortening the transient process and saving resources. On the basis of the designed memory event triggering scheme, some sufficient conditions for the exponentially ultimately bounded of complex dynamic networks are derived according to Lyapunov function and linear matrix inequalities. Finally, the validity of the theoretical results is verified by a numerical simulation case.

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