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Anisotropic Cosmological Model with Quark and Strange Quark Matter in $f(R,T)$ Gravity Theory

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Abstract Einstein's field equations in $f(R,T)$ gravity theory in the presence of quark and strange quark matter are considered for a plane symmetric space-time. We have obtained exact solutions of the field equations by assuming a special form of Hubble parameter that yield a time-varying deceleration parameter. The physical behaviors of the cosmological model are discussed.

Keywords: Plane symmetric Space-Time; Quark and Strange Quark Matter; $f(R,T)$ Gravity Theory; Time-Dependent Deceleration Parameter.

1 Introduction

The most remarkable observational discoveries in cosmology have shown that our current universe is not only expanding but also accelerating. This was first observed from high red shift supernova I_a (Riess et al. [1,2]; Perlmutter et al. [3]; Astier et al. [4]) and later confirmed by cross checks from microwave background radiation (Bennet et al. [5]; Spergel et al. [6] Tegmark et al. [7]). In Einstein's general relativity in order to have such acceleration one needs to introduce a component to the matter distribution of the universe with a large negative pressure. This component is referred as dark energy (DE). The limitations of general relativity in providing satisfactory explanation of this phase of evolution have led cosmologists to modify Einstein's general relativity. Modified gravity theories certainly provide a way of understanding the problem of DE and possibly to reconstruct gravitational field theories that would be capable to reproduce the late-time acceleration of the universe. Among several modified theories of gravitation, $f(R,T)$ gravity theory proposed by Harko et al. [8] is widely accepted. Here R is the Ricci scalar and T is the trace of energy momentum tensor $T_{\mu\nu}$. As $f(R,T)$ gravity depends on matter field, different theoretical models corresponding to specific matter distributions could be derived. Three classes of models have been obtained for the following choices of the function $f(R,T)$:

$$\begin{aligned} f(R,T) &= R + 2f(T) \\ &= f_1(R) + f_2(T) \\ &= f_1(R) + f_2(R) f_3(T). \end{aligned} \tag{1}$$

Several cosmologists have presented so for cosmological model in $f(R,T)$ gravity theory for above forms of $f(R,T)$ in different context.

In quantum field theories broken symmetries are restored at very high temperatures. It is well known that quark gluon plasma existed during one of the phase transitions of the universe at the early time when the cosmic temperature $T \sim 200MeV$. One of the interesting consequences of the first order phase transition from the quark phase to hadron phase in the early universe is the formation of strange quark matter. Quark matter has drawn much interest (Witlen [9]; Fahri and Jaffe [10]). Typically, strange quark matter is developed with the equation of state $p = \frac{1}{3}(\rho - 4B_c)$ based on the phenomenological bag model of quark matter in which quark confinement is described by an energy term proportional to the volume. In this equation B_c, being the difference between the energy density of the perturbative and nonperturbative QCD vacuum, is known as bag constant. Here ρ and p are the energy density and thermodynamic pressure of quark matter respectively. In this model quarks are thought a degenerate Fermi gas which exists only in a region of space endowed with vacuum energy density B_c. In the framework of this model, the quark matter is composed of massless u and d quarks, massive quarks and electrons. In the simplified version of the bag model, it is assumed that quarks are massless and noninteracting. Therefore, the quark pressure $p_q = \frac{\rho_q}{3}$ where ρ_q is the quark energy density. Hence, the total energy density is $\rho = \rho_q + B_c$ and the total pressure is $p = p_q - B_c$.

The study of quark and strange quark matter has drawn the attentions of many researchers. Mak and Harko [11] have studied charged strange quark matter in a spherically symmetric space-time admitting conformal motion. Yavuz et al. [12] have studied the strange quark matter attached to string cloud in spherically symmetric space-time admitting conformal motion. Adhav et al. [13,14] have studied string cloud and domain walls with quark matter in an n-dimensional Kaluza-Klein cosmological model in general relativity and strange quark matter attached to a string cloud in Bianchi type III space-time. Yilmaz et al. [15] have discussed quark and strange quark matter within the framework of $f(R)$ gravity for Bianchi type I and V space-times. Katore [16] has studied a cosmological model with strange quark matter attached to cosmic strings in FRW models. Khadekar and Salote [17] have obtained higher dimensional cosmological model in the presence of quark and strange quark matter. Sahoo and Mishra [18] have studied axially symmetric cosmological model with string cloud universe containg strange quark matter. Further Adhav et al. [19] have discussed Kantowski-Sachs cosmological model with quark and strange quark matter in $f(R)$ gravity theory. Recently, Agrawal and Pawar [20] have studied plane symmetric cosmological model with quark and strange quark matter in $f(R,T)$ gravity theory by applying a special law of variation of Hubble parameter that yields a constant value of deceleration parameter.

In this paper, we investigate a new plane symmetric cosmological model in the presence of quark and strange quark matter with in the framework of $f(R,T)$ gravity theory. The paper is organized as follows: In Sect. 2, we present the metric and field equations. In Sect. 3, we obtain exact solutions of the field equations by assuming a special form of Hubble parameter that yield a time-dependent deceleration parameter. The physical properties of the plane symmetric cosmological model are discussed in Sect. 4. Finally, some concluding remarks are given in Sect. 5.

2 Metric and Field Equations

We consider the metric of the plane symmetric space-time of the form

$$ds^2 = dt^2 - A^2\left(dx^2 + dy^2\right) - B^2 dz^2 \tag{2}$$

where A and B are cosmic scale factors and are functions of comic time t.

The gravitational field equations of $f(R,T)$ gravity theory with the first form $f(R,T)$ in equation (1) are given by

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu} + 2f^{'}(T) + \left[2pf^{'} + f(T)\right]g_{\mu\nu} \tag{3}$$

if the matter source is a perfect fluid, where a prime denotes differentiation with respect to the argument. This gravity theory is equivalent to a cosmological model with an effective cosmological term $\Lambda \sim H^2$ [21].

The energy-momentum tensor for quark matter is given as

$$T_{\mu\nu}^{(\text{Quark})} = (\rho + p)\,u_\mu u_\nu - pg_{\mu\nu} \tag{4}$$

where $\rho = \rho_q + B_c$, $p = p_q - B_c$ and $u^\mu = \delta_0^\mu$ is the four velocity in comoving coordinates. Since quark matter behaves as a nearly perfect fluid, we take the following equation of state for quark matter

$$p_q = \epsilon\rho_q, \qquad 0 \le \epsilon \le 1. \tag{5}$$

Also the linear equation of state for strange quark matter is [22, 23]

$$p = \epsilon\left(\rho - \rho_0\right) \tag{6}$$

where ρ_0 is the energy density at zero pressure and ϵ is a constant. When $\epsilon = \frac{1}{3}$ and $\rho_0 = 4B_c$, the above linear equation of state is reduced to the following equation of state for strange quark matter in the bag model [23]

$$p = \frac{\rho - 4B_c}{3} \tag{7}$$

where B_c is the bag constant.

In comoving coordinates system, the field equation (3) for the metric (1) with help of (4) and assuming $f(T) = \lambda T$ can be written as [20]

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} = p_q - B_c - \lambda\left(\rho_q + 4B_c - 3p_q\right), \tag{8}$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\ddot{A}}{A} = p_q - B_c - \lambda\left(\rho_q + 4B_c - 3p_q\right), \tag{9}$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} = -\rho_q - B_c - \lambda\left(3\rho_q + 4B_c - p_q\right) \tag{10}$$

We define the spatial volume V and average scale factor a of the universe as

$$V = A^2 B, \qquad a = \left(A^2 B\right)^{1/3}. \tag{11}$$

The directional Hubble parameters in the directions of x, y and z axes respectively are defined as

$$H_1 = H_2 = \frac{\dot{A}}{A}, \qquad H_3 = \frac{\dot{B}}{B}. \tag{12}$$

The mean Hubble parameter H is defined as

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(\frac{2\dot{A}}{A} + \frac{\dot{B}}{B}\right). \tag{13}$$

The expansion scalar θ and shear scalar σ^2 are given by

$$\theta = 3H, \qquad \sigma^2 = \frac{2}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2. \tag{14}$$

The anisotropy parameter A_m is defined by

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{15}$$

The deceleration parameter q is defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{16}$$

The positive sign of q corresponds to standard decelerating model, whereas the negative sign accelerated expansion.

3 Solution of the Field Equations

Subtracting Equation (8) from Equation (9), we obtain

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0. \tag{17}$$

This equation can be integrated to give

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k}{V}, \tag{18}$$

k being an arbitrary constant of integration.

In order to obtain a model of the universe consistent with recent observations, one has to define the Hubble parameter such that the model starts with a decelerated expansion followed by an accelerated expansion at late times. Following Ellis and Madsen [24], Singh [25] has defined a functional form of H as

$$H = \frac{\dot{a}}{a} = \alpha\left(1 + a^{-n}\right) \tag{19}$$

which despite being quite simple, describes both decelerating and accelerating phases of the universe where $\alpha(>0)$ and $n(>1)$ are constants. Solving Equation (19), one can obtain the average scale factor as

$$a = \left(e^{\alpha n t} - 1\right)^{\alpha/n}. \tag{20}$$

Adhav et al. [26] utilized the above form of the average scale factor to study spatially homogeneous and anisotropic Bianchi types I, III, V, VI_0 and Kantowski-Sachs space-times with variable equation of state parameter in general relativity. From Equation (20), one can easily derive the deceleration parameter of the form

$$q = -1 + \frac{n}{1 + a^n}. \tag{21}$$

From Equation (11), we obtain

$$\frac{2\dot{A}}{A} + \frac{\dot{B}}{B} = \frac{\dot{V}}{V}. \tag{22}$$

Solving Equation (18) and (22) and using Equation (21), we get

$$\frac{\dot{A}}{A} = \frac{k}{3V} + \frac{\dot{V}}{3V} = \frac{k}{3\left(e^{\alpha n t} - 1\right)^{3\alpha/n}} + \frac{\alpha^2 e^{\alpha n t}}{\left(e^{\alpha n t} - 1\right)}, \tag{23}$$

$$\frac{\dot{B}}{B} = \frac{\dot{V}}{3V} - \frac{2k}{3V} = \frac{\alpha^2 e^{\alpha n t}}{\left(e^{\alpha n t} - 1\right)} - \frac{2k}{3\left(e^{\alpha n t} - 1\right)^{3\alpha/n}}. \tag{24}$$

For simplification in integration of (23) and (24), we take $\alpha = 1$ and $n = 3$, so that

$$\frac{\dot{A}}{A} = \frac{k}{3\left(e^{3t} - 1\right)} + \frac{e^{3t}}{\left(e^{3t} - 1\right)}, \tag{25}$$

$$\frac{\dot{B}}{B} = \frac{e^{3t}}{\left(e^{3t} - 1\right)} - \frac{2k}{3\left(e^{3t} - 1\right)}. \tag{26}$$

Solutions of Equations (25) and (26) are given by

$$A = c_1 e^{-\frac{kt}{3}} \left(e^{3t} - 1\right)^{\frac{(k+3)}{9}}, \tag{27}$$

$$B = c_1^{-2} e^{\frac{2kt}{3}} \left(e^{3t} - 1\right)^{\frac{(3-2k)}{9}}. \tag{28}$$

where c_1 is an integration constant. Without loss of generality, we take $c_1 = 1$. Hence, the metric of our solution can be written as

$$ds^2 = dt^2 - e^{-\frac{2kt}{3}} \left(e^{3t} - 1\right)^{\frac{2(k+3)}{9}} \left(dx^2 + dy^2\right) - e^{\frac{4kt}{3}} \left(e^{3t} - 1\right)^{\frac{2(3-2k)}{9}} dz^2. \tag{29}$$

4 Some Physical Properties

Now we discuss some physical features of the plane symmetric cosmological model (29). The directional Hubble parameters in the directions of x, y and z axes are given by

$$H_1 = H_2 = \frac{k}{3\left(e^{3t} - 1\right)} + \frac{e^{3t}}{\left(e^{3t} - 1\right)}, \tag{30}$$

$$H_3 = \frac{e^{3t}}{(e^{3t}-1)} - \frac{2k}{3\,(e^{3t}-1)}. \tag{31}$$

The mean Hubble parameter H is obtained by using Equations (30) and (31) as

$$H = \frac{e^{3t}}{(e^{3t}-1)}. \tag{32}$$

The expansion scalar $\theta = 3H$ has the value

$$\theta = \frac{3e^{3t}}{(e^{3t}-1)}. \tag{33}$$

The mean anisotropy parameter of the expansion is found to be

$$A_m = \frac{k^2}{9e^{6t}}. \tag{34}$$

The shear scalar has the value given by

$$\sigma = \frac{k}{(e^{3t}-1)}. \tag{35}$$

The deceleration parameter as calculated from Equation (21) is

$$q = -1 + 3e^{-3t}. \tag{36}$$

The expansion scalar and shear scalar are infinite at the initial singularity and are decreasing functions of time. At $t = 0$, the anisotropy parameter is finite equals $\frac{k^2}{9}$ and decreases with time, and ultimately tends to zero as $t \to \infty$. The expansion scalar tends to constant value where as σ tends to zero as $t \to \infty$. Also the ratio $\frac{\sigma}{\theta}$ tends to zero as $t \to \infty$. these indicate that the model was highly anisotropic at the early stages and approaches isotropy for large time.

The deceleration parameter is positive for $t < \frac{1}{3}\log 3$ and is negative for $t > \frac{1}{3}\log 3$. This means that the universe starts evolving from $t = 0$ with decelerating expansion for a short time and the expansion changes from deceleration to acceleration at time $t = \frac{1}{3}\log 3$. This shows that q has a signature flip at this epoch. The signature flip in q is an essential for the conclusion that the present universe is accelerating. Thus, the model leads to a cosmological scenario in accordance with the well known features of modern cosmology as an initial decelerating phase is followed by accelerating one at late time. From Equation (36) we observe that $q = -1$ as $t \to \infty$. This means that the universe asymptotically achieves the de Sitter phase and hence expands forever.

Subtracting Equation (9) from Equation (10), we obtain

$$\frac{2\ddot{A}}{A} - \frac{\dot{A}}{A}\frac{\dot{B}}{B} = (1+2\lambda)\,(\rho_q + p_q) \tag{37}$$

Substituting the values of A and B from Equations (27) and (28) and using the equation of state ($p_q = \epsilon\rho_q$) for $\epsilon = \frac{1}{3}$, we obtain the quark matter density and quark pressure as

$$\rho_q = \frac{k^2 - 9e^{3t}}{6(1+2\lambda)(e^{3t}-1)^2} \tag{38}$$

$$p_q = \frac{\left(k^2 - 9e^{3t}\right)}{18(1+2\lambda)(e^{3t}-1)^2}. \tag{39}$$

Using Equations (27), (28), (37) and using the equation of state in Equation (7), we find the enery density and pressure of the strange quark matter as follows,

$$\rho = \frac{k^2 - 9e^{3t}}{18(1+2\lambda)(e^{3t}-1)^2} + B_c \tag{40}$$

$$p = \frac{\epsilon\left(k^2 - 9e^{3t}\right)}{6(1+2\lambda)(e^{3t}-1)^2} - B_c \tag{41}$$

5 Conclusions

In this paper we have studied a plane symmetric cosmological model with quark matter and strange quark matter in $f(R,T)$ gravity theory. We have obtained exact solutions of the field equations by assuming the time-dependent from of the average scale factor that yields a time-varying deceleration parameter. We have evaluated the matter density and thermodynamic pressure for quark matter and strange quark matter.

The spatial volume of this model is zero at $t = 0$. At this epoch the pressure p_q and energy density ρ_q for quark matter are infinite and as $t \to \infty$, they tend to zero. The pressure p_q and energy density $\rho_q a$ of strange quark matter behave in the same way as that of quark matter, except it is shifted by $\epsilon\rho_0$.

It can be seen that the expansion scalar θ starts with infinite value at $t = 0$, and as time increases, it decreases to a constant value and remains constant as $t \to \infty$. The universe is anisotropic at $t = 0$ and approaches to isotropy at late times. The deceleration parameter $q = -1$ as $t \to \infty$ shows that the universe asymptotically achieves the de Sitter phase and expands forever. The results obtained in this paper may be useful in the study of structure formation of the universe . This cosmological model describes the early decelerating and late-time accelerating universe which is a special characteristic of the present day expanding universe. It deserves to mention that the assumption of time-dependent average scale factor in equation (19) is not based on any known theory. However, it provides an appropriate description of the universe consistent with observation.

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References

1. A. Riess et al., "Observational Evidences from Supernovae for an Accelerating Universe and a Cosmological Constant", *Astronomial Journal*, Vol.116, 83-88, 1998.
2. A. Riess et al., "Type I$_a$ Supernova Discoverics at $z > 1$ from the Hubble Space Telescope: Evolution for Past Deceleration", *Astrophysical Journal*, Vol. 607, 665-687, 2004.
3. S. Perlmutter et al., "Measurements of Ω and Λ from 42 High Redshift Supernovae", *Astrophysical Journal*, Vol. 517, 515-586, 1999.
4. P. Astier et al., "The Supernova Legacy Survey: Measurement of ω_m, ω_Λ and ω from the First year Data Set", *Astronomy and Astrophysics*, Vol. 447, 31-48, 2006.
5. C. L. Bennet et al, "The Third Data Release of the Sloan Sky Survey", *Astrophysical Journal Supplement Ser*, Vol. 148, 97-117, 2003.
6. D. N. Spergel et al., "First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters", *Astrophysical Journal* , Vol. 148, 175-194, 2003.
7. M. Tegmark et al., "Cosmological Parameter from SDSS and WAMP", *Physical Review D*, Vol. 69, 107501, 2004.
8. T. Harko et al., "f(R,T) Gravity", *Physical Review D*, Vol. 84, 024020, 2011.
9. E. Witten, "Cosmic Separation of Phases", *Physical Review D*, Vol.30, 272, 1984.
10. E. Farhi, R. L. Jaffe, "Strange Matter"*Physical Review D*, Vol.30, 2379, 1984.
11. M. K. Mak, T. Harko, "Quark Stars Admitting a One-Parameter Group of Conformal Motions", *International Journal of Modern Physics D*, Vol.13, 149-156, 2004.
12. I. Yavuz, I. Yilmaz, H. Bayasal, "Strange Quark Matter Attached to the String Cloud in the Spherical Symmetric Space-time",*International Journal of Modern Physics D*, Vol.14, 1365-1372, 2005.
13. K. S. Adhav et al.,"Kantowski-Sachs Cosmological Model in General Theory of Relativity", *International Journal of Theoretical Physics* , Vol.47, 2314-2318, 2008.
14. K. S. Adhav et al.,"Strange Quark Matter Attached to String Cloud in Bianchi-III Space-Time", *Astrophysics and Space Science* , Vol.319, 81-84, 2009.
15. I. Yilmaz, H. Bayasal, C. Aktas, "Quark and Strange Quark Matter f(R) Gravity for Bianchi I and V Space-times", *General Relativity and Gravitation*, Vol. 44, 2313-2328, 2012.
16. S. D. Katore, "Strange Quark Matter Attached to String Cosmology in FRW Space-Time "*International Journal of Theoretical Physics* , Vol.51, 83-89, 2012.
17. G. S. Khadekar, R. Shelote, "Higher Dimensional Cosmological Model with Quark and Strange Quark Matter", *International Journal of Theoretical Physics* , Vol.51, 1442-1447, 2012.

18. P. K. Sahoo, B. Mishra, "Axially Symmetric Space-Time with Strange Quark Matter Attached to String Cloud in Bimetric Theory", *International Journal of pure and Applied Mathematics*, Vol.82, 87-94, 2013.
19. K. S. Adhav et al., "Kantowski-Sachs Cosmological Model with Quark and Strange Quark Matter in f(R) Theory of Gravity", *Open Physics*, Vol. 13, 90-95, 2015.
20. P. K. Agrawal, D. D. Pawar, "Plane Symmetric Cosmological Model with Quark and Strange Quark Matter in f(R, T) Theory of Gravity", *Journal of Astrophysics and Astronomy*, Vol. 38, 2-7, 2017.
21. N. J. Poplawski, "A Lagrangian Description of Interacting Dark Energy", *arXiV:gr-qc,*, 0608031, 2006.
22. B. B. Back et al., "First Three Years of Operation of RHIC", *Nuclear Physics A*, Vol. 757, 1-284, 2005.
23. C. Aktas et al., "Is the Universe Homogeneous and Isotropic in the Time When Quark-Gluon Plasma Exits", *General Relativity and Gravitation*, Vol. 43, 1577-1591, 2011.
24. G. F. R. Ellis, M. Madsen,"Exact scalar field cosmologies", *Classical and Quantum Gravity*, Vol. 8, 667-676, 1991.
25. J. P. Singh, "A Cosmological Model with both Deceleration and acceleration",*Astrophysics and Space Science*, Vol.318, 103-107, 2008.
26. K. S. Adhav et al.,"Early Decelerating and Late-Time Acceleration Anisotropic Cosmological Models with Dynamical EoS Parameters", *Astrophysics and Space Science*, Vol.345, 405-413, 2013.

